(184)

3

(182)

-: HAND WRITTEN NOTES:-

OF

(1)

ELECTRONICS & COMMUNICATION ENGINEERING

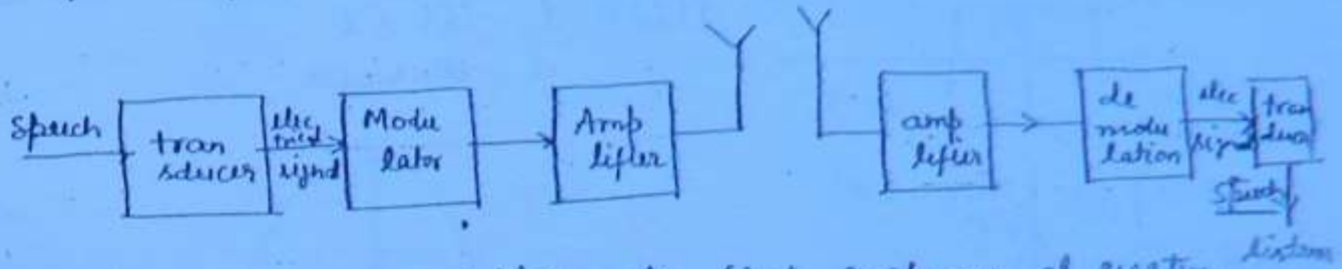
-: SUBJECT:-

SIGNAL & SYSTEM

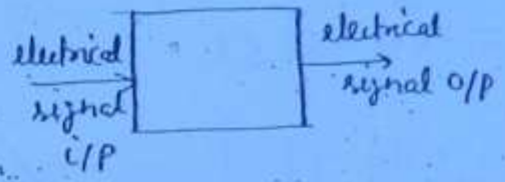
3

2

Speech signal → 300Hz → 3400Hz



* SS → we ~~ar~~ are concentrating to find response of system



to find response of a system, we use following rules/tools-

1. Fourier series
2. Fourier transforms
3. Laplace transforms
4. Z- transforms

Audio signal → 20Hz - 20KHz

Video signal → 0 - 5MHz.

data signal → range is based on application

Signal → is a quantity having associated information with it.

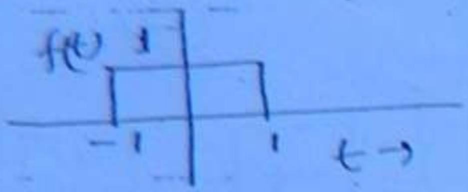
is a function of time.

we deal with electrical signal which are voltages or currents, which are both functions of time. In general signal is a function time f(t). (electrical)

But signal is not always a function of time

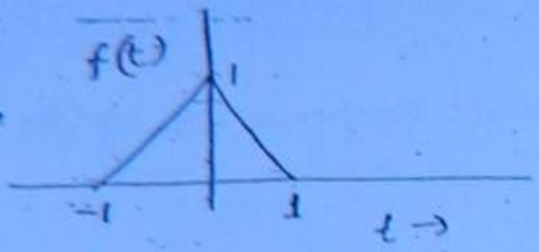
collect still frame and play back to it is video signal (normally at 24 frames/second where play back the still frames).

In Motion picture signal is three dimensional f(x, y, t



$f(t) = 1 \quad -1 \leq t \leq 1$

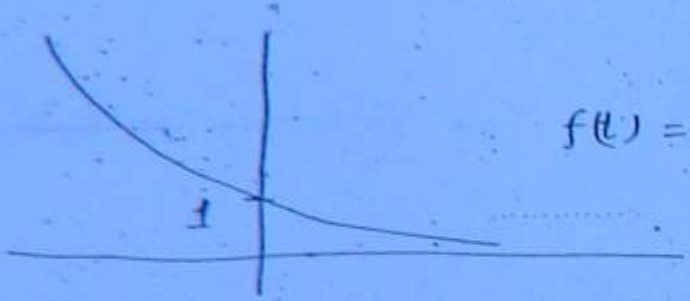
$0 \quad \text{otherwise}$



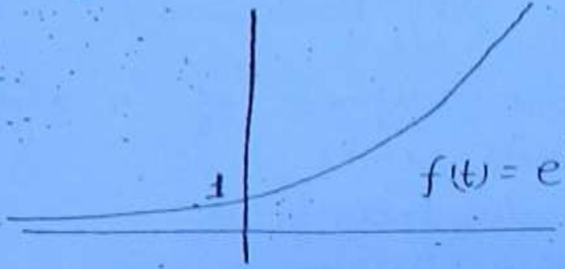
$f(t) = \quad -1 \leq t \leq 0$

$= t+1$

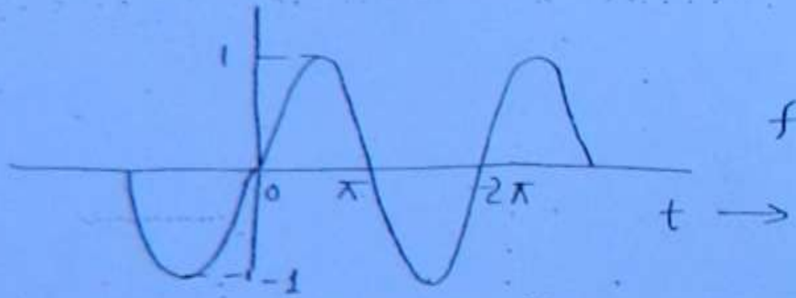
$= -t+1 \quad 0 \leq t \leq 1$



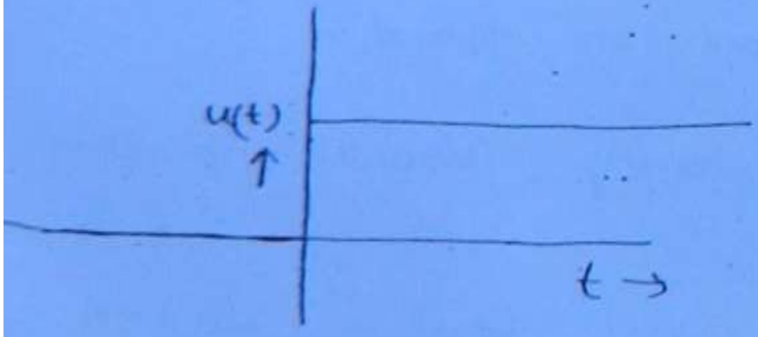
$f(t) = e^{-at}$



$f(t) = e^{at}$



$f(t) = \sin t$



$u(t) = 1 \quad t \geq 0$

$0 \quad t < 0$

unit impulse signal

existence is split second
effect is anormous.

$\delta(t) = 0 \quad t \neq 0$

$= 0 \quad t = 0$

$\neq 0 \quad t = 0$ (very large ∞)

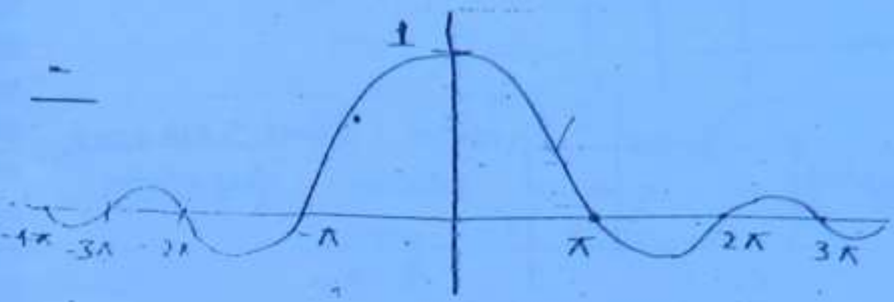
rect $\cdot \frac{(t-1)}{2}$

3/4

rect(t + 3/4)

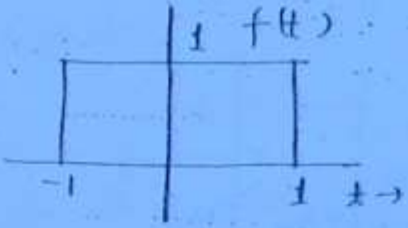
$$\int_{-\infty}^{\infty} f(t)\,dt = 1 \rightarrow \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

⑤

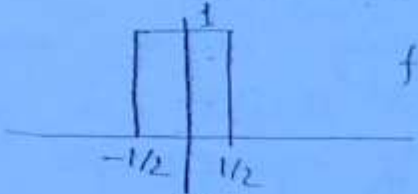


$f(t) = \frac{\sin t}{t} = Sa(t)$

$= \mathrm{Sinc}(t/\pi)$



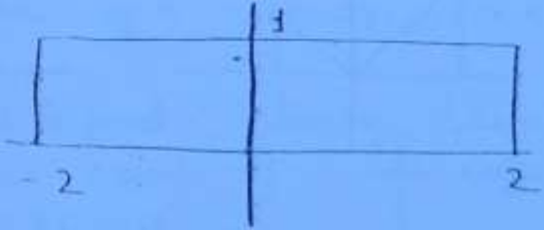
$f(t) = \mathrm{rect}(\frac{t}{2})$



f(2(t))

f(t) = rect(t)

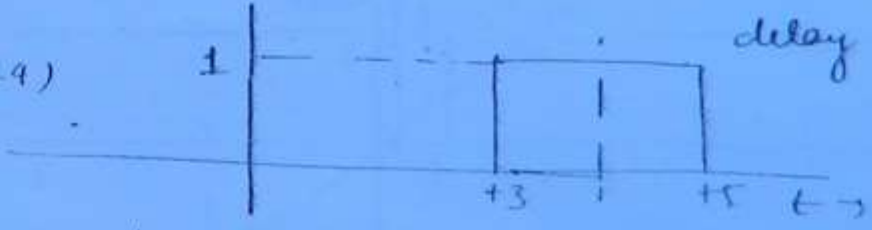
time scaled version of f(t) (amplified time-axis change)



f(t/2)

f(t) = rect(t/4)

rect(t − 4)



delay

time shifted version of given signal f(t)

f(t − 4)

rect(t + 3)

advanced

f(t + 3)

[(divide all time(instant) by a)] $f(t) \xrightarrow[\text{(time-scaling)}]{} f(at)$ $a > 1 \rightarrow$ compress

$0 < a < 1 \rightarrow$ expansion

⑥

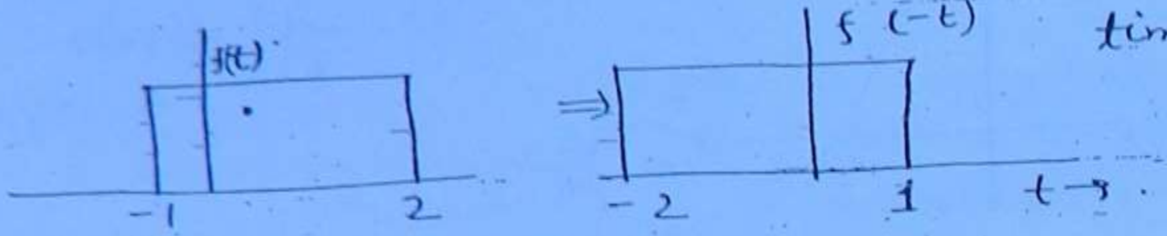
↗ (subtract T from all instant) shift

$f(t) \xrightarrow{t \to (t+T)}$ → advanced (left)

shift

$t_2(t-T)$ → delayed (right)

↳ add (T to all instant)



time reversal operation

$f(t) = 1 \quad -1 \leq t \leq 2$

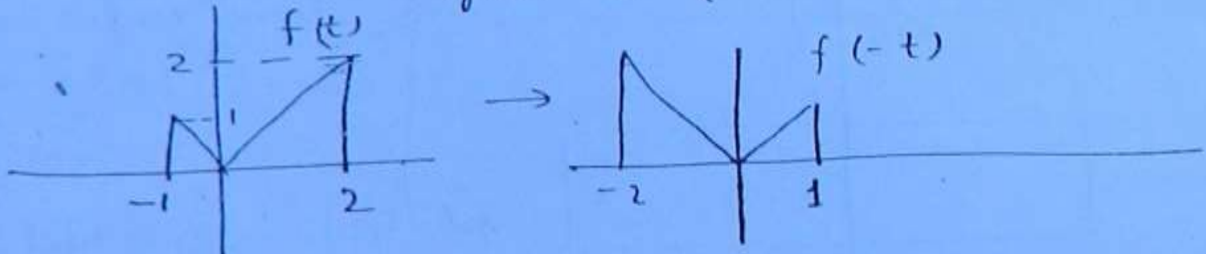
$\quad 0 \quad$ otherwise

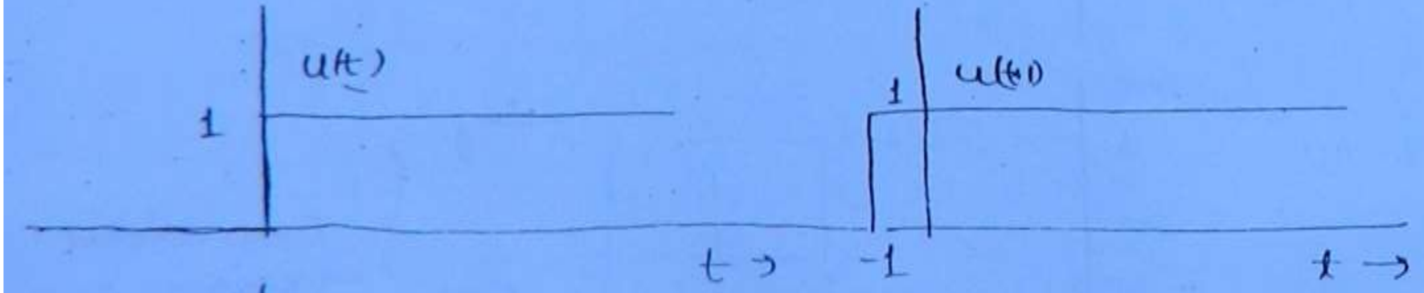
$f(-t) = \quad -1 \leq -t \leq 2$

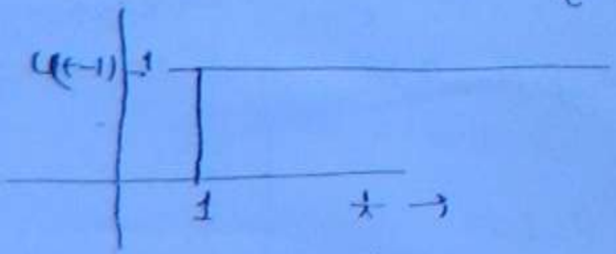
$\quad = \quad 1 \geq t \geq -2$

$\quad 0$ otherwise

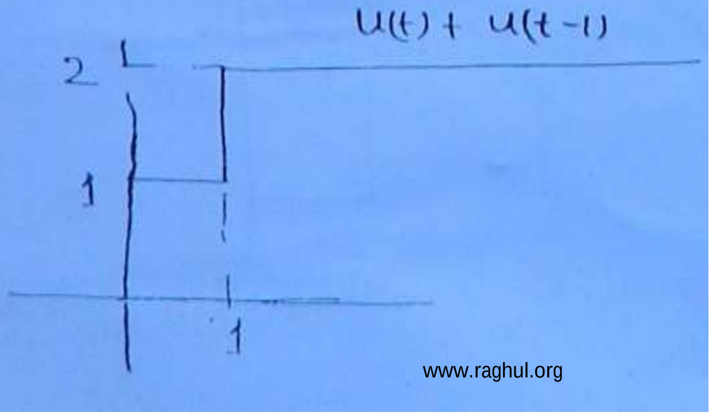
obtained by rotating signal across Y axis / by 180° or by taking mirror image of signal abt Y-axis.



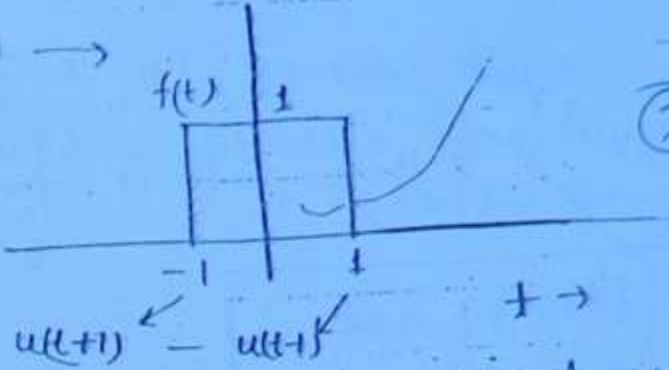




u(t) + (u t-1) →

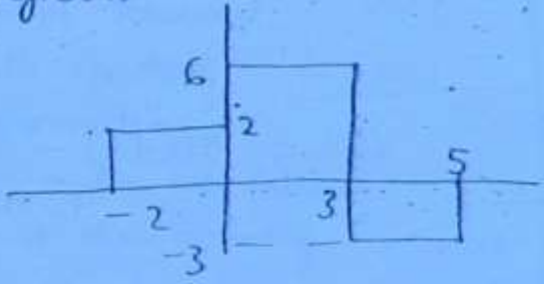


$f(t) = u(t+1) - u(t-1) \rightarrow$



$u(t) \rightarrow$ indicates change in signal value from 0 to 1 exactly at $t = 0$

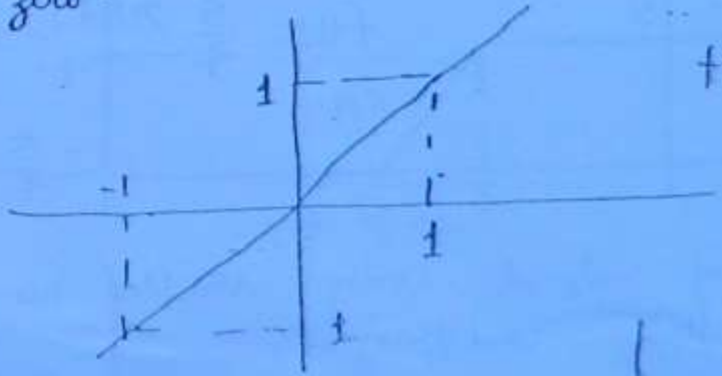
Q. represent the following signal using shifted unit step signals.



$2u(t+2) + 4\,u(t)$
$- 9\,u(t-3)$
$+ 3\,u(t-5)$

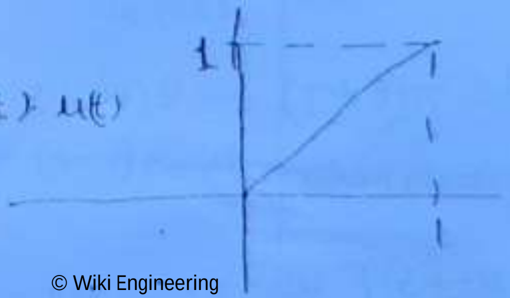
as long as signal is nonzero only for a finite inter of time then some of coefficient will be zero as in above eq → $2+4-9+3 = 0$

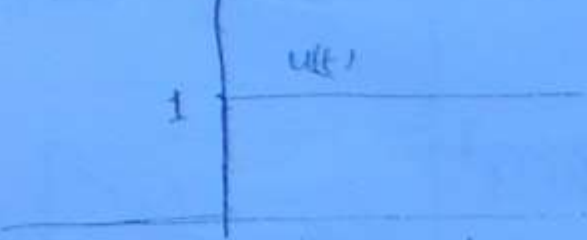
(*) A signal having step changes in finite interval time then some of all cofficient of shifted unit step signal will be zero.



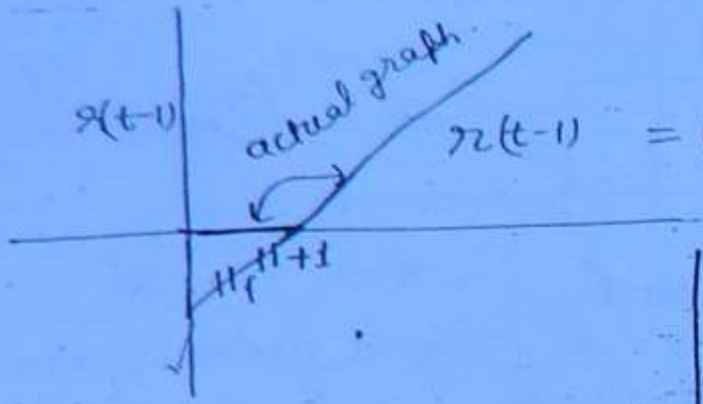
$f(t) = t$ [not ramp signal]

$r(t) \cdot u(t)$



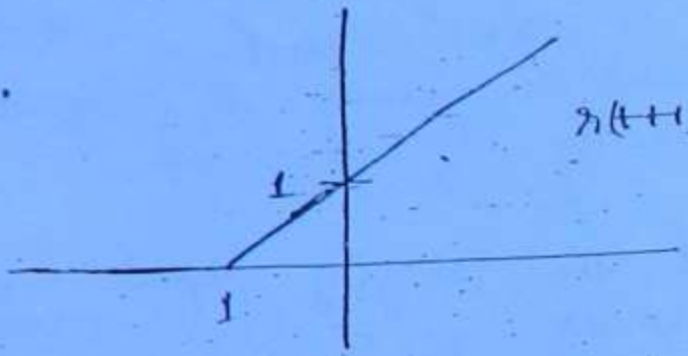


$f(t) = t \quad 0 \leq t \leq \infty$ or $t \geq 0$ ramp signal
0 otherwise



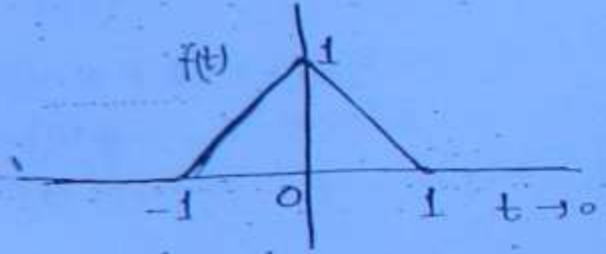
$r(t-1) = (t-1)$ for $t \geq 1$

0 $t<1$



$r(t+1) = (t+1)$ for $t \geq -1$

0 otherwise

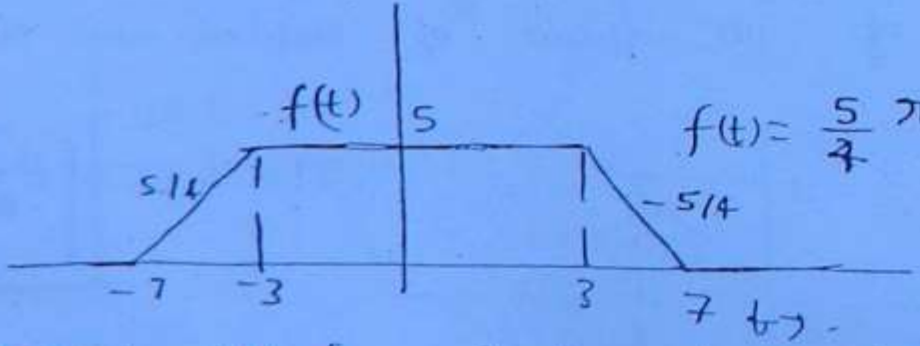


$$f(t) = r(t+1) - 2\,r(t) + r(t-1)$$

r(t) represent change in slope at t=0 from $\overset{\text{slope}}{(0 \to 1)}$

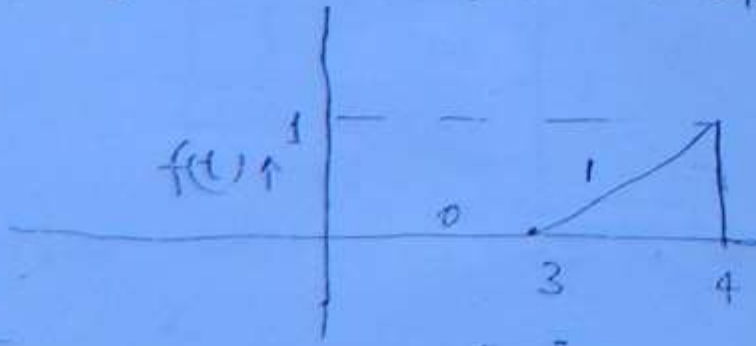
if signal is finite interval signal some of coefficients will be zero → eg $1-2+1 = 0$

Q. Represent following signal using shifted ramp signal.



$$f(t) = \frac{5}{4} r(t+7) - \frac{5}{4} r(t+3) - \frac{5}{4} r(t-3) + \frac{5}{4} r(t+7)$$

Q. represent following signal using shifted ramp signals and shifted unit signals.



$f(t) = r(t+3) - u(t-4)$

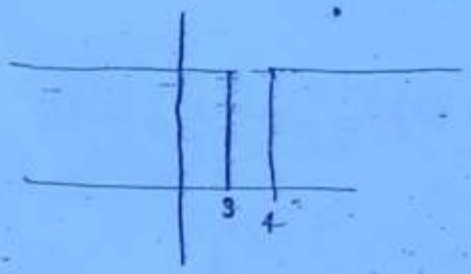
sawtooth pulse $+ r(t-4)$

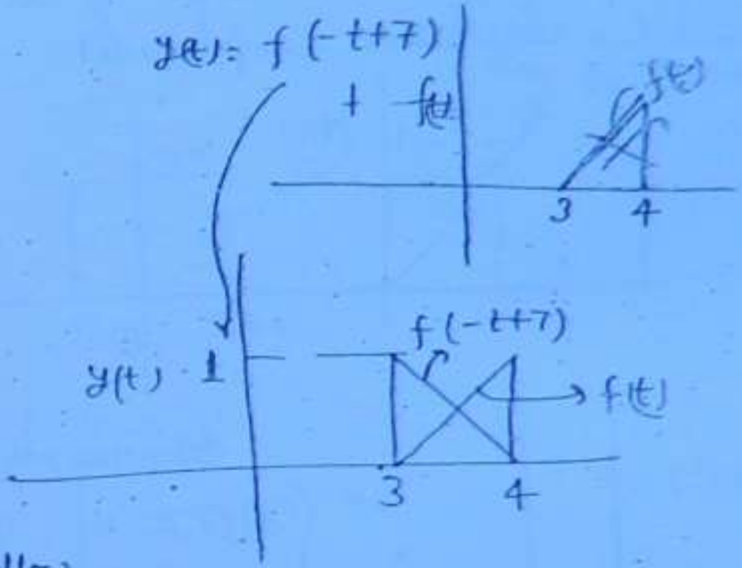
$r(t+3)[u(t-3) - u(t-4)]$

(ii) for the above signal f(t) + f(-t+7) will be

$$f(t)+f(-t+7) = r(t-3) - r(t-4) - u(t-4) + r(-t+4) - r(-t+3) - u(-t+3)$$

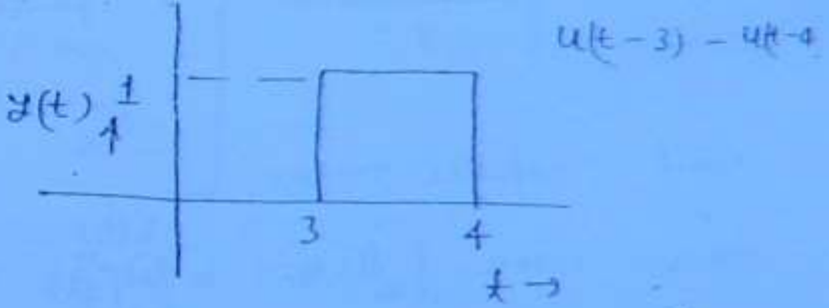
=





If shifting and reversal operation both involved in given transformation the normal order we follow shifting first and then reversing but we can also do this process in

f(-t+7) reverse order.



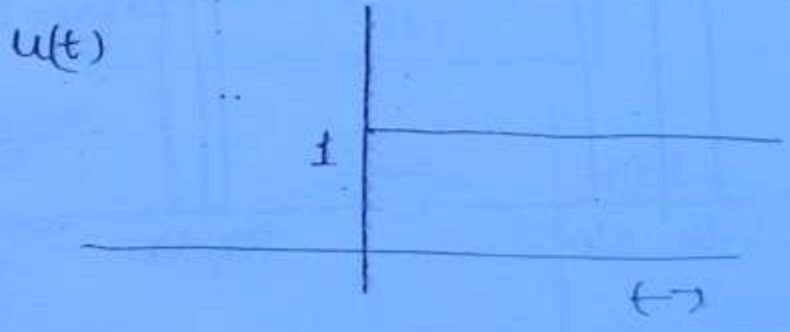
$$f(t) \xrightarrow{\text{reversal}} f(-t)$$

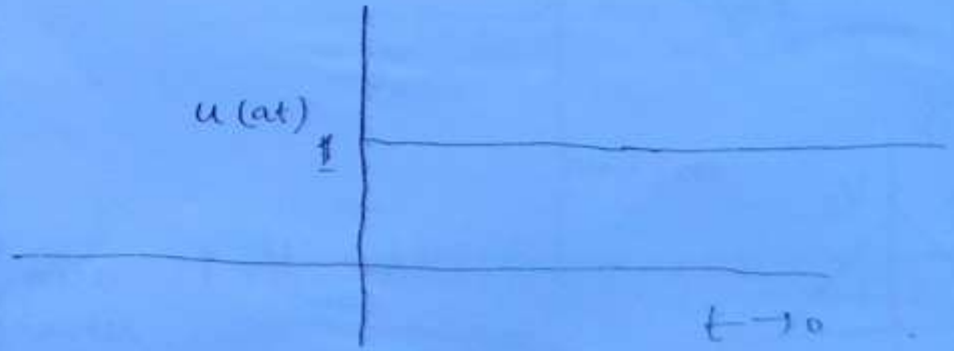
$$\text{shift} \xrightarrow[\text{delayed}]{\text{by } 7} f(-t+7)$$

$f((-)[t-7])$

$f(at+b)$

$f(a[t+b/a])$

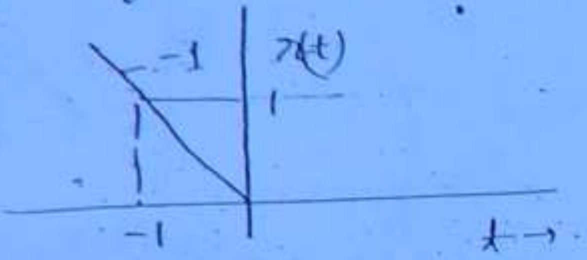




$x(at) =$ $at \quad at \geq 0$
$0 \quad at < 0$ $\longrightarrow \left\{ a > 0 \right\}$

$x(at) = a\,x(t)$

* in ramp signal scaling of time results in scaling of magnitude (amplitude).



$x(-t) = -t \quad t \leq 0$
$\qquad\quad 0 \quad t > 0$

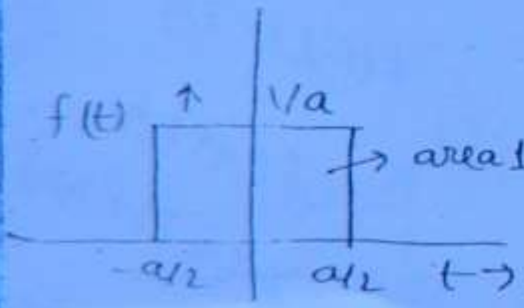
definition of delta function

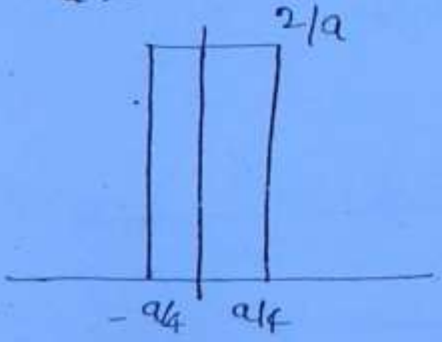
$$\begin{cases} \delta(t) = 0 & t \neq 0 \\ \quad \neq 0 & t = 0 \\ \int_{-\infty}^{\infty} \delta(t)\,dt = 1 \end{cases}$$

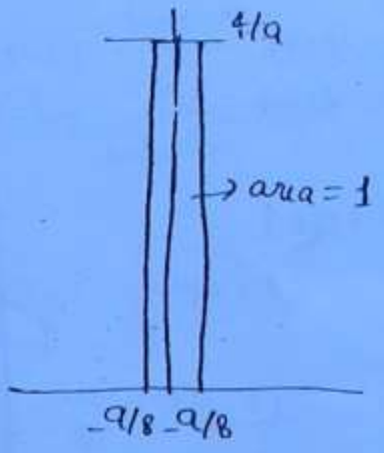
. also called dirac delta function.

unit impulse means
↓
means area $\int_{-\infty}^{\infty} \delta(t) dt = 1$ $= \lim_{a \to 0} f''(t)$

$\delta(t) = \lim_{a \to 0} f(t)$

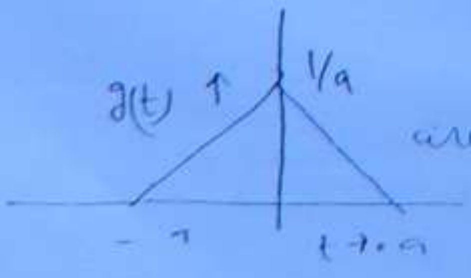






$a \to 0$ hight $\longrightarrow \infty$

or

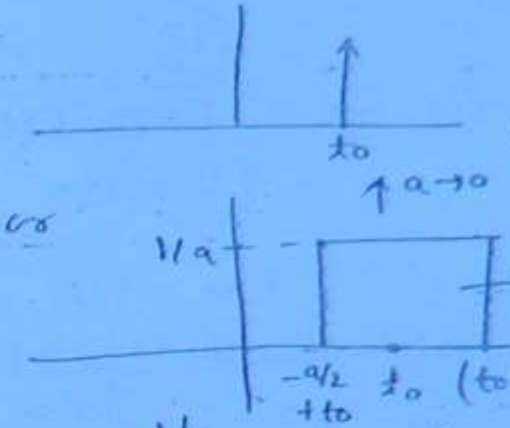


area = 1

a ↓ 1/a ↑ area remains same

$\delta(t) = \lim_{a \to 0} g(t)$

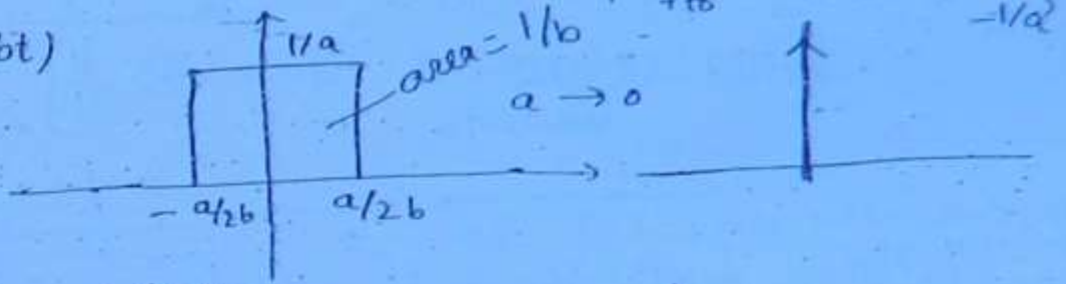
$\delta(t-t_0) = \lim_{a\to 0} f(t-t_0) =$



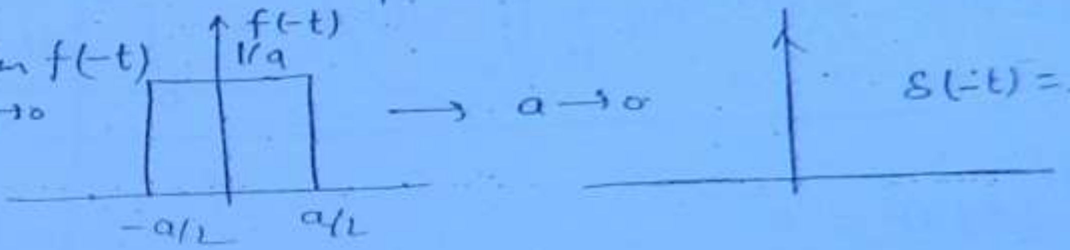
$\delta(bt) = \lim_{a\to 0} f(bt)$

$\frac{1}{b}\delta(t)$

$= 1/_b\, \delta(t)$



$\delta(-t) = \lim_{a\to 0} f(-t)$



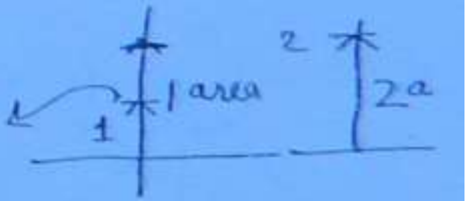
$\delta(-t) = \delta(t)$

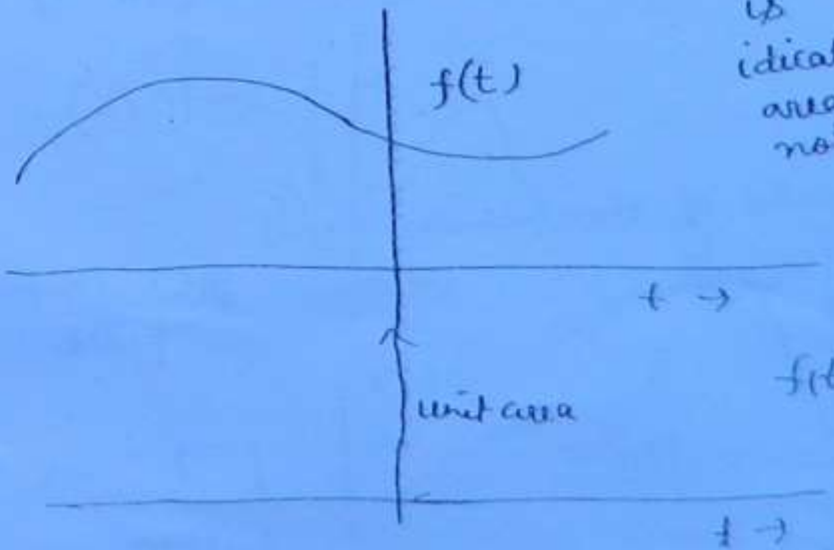
$\delta(-2t) = \frac{1}{2}\delta(t)$

$\delta(-2t) = \delta(2t) = 1/_2\, \delta(t)$

$$\boxed{\delta(bt) = \frac{1}{|b|}\delta(t)}$$

hight is idicating area not the value





$f(t)\cdot\delta(t) = f(0)\cdot\delta(t) = \delta(t)$ (If $f(0) > 0$, … is …

amplitude ∞ but effect on area
area will become
$= f(a)$, b/w

$f(t)\,\delta(t-a) = \overset{\text{amplitude }\infty}{\overline{f(a)\cdot\delta(t-a)}}$ → sampling Property of impulse signal. $\left(\begin{smallmatrix}-\infty \text{ to } \infty\\ \text{curve}\end{smallmatrix}\right)$

$= f(a)\cdot\delta(t-a)$

$$\int_{-\infty}^{\infty} f(t)\,\cancel{dt}\,\delta(t)\,dt = f(0)\int_{-\infty}^{\infty}\delta(t)\,dt = f(0)$$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\,dt = f(a) = \int_{-\infty}^{\infty} f(a)\,\delta(t-a)\,dt$$

$$= f(a)\int_{-\infty}^{\infty}\delta(t-a)\,dt = f(a)$$

$$\boxed{\int_{-\infty}^{\infty}\delta(t)\,dt = \int_{-1}^{1}\delta(t)\,dt = \int_{-a}^{a}\delta(t)\,dt = \int_{0^-}^{0^+}\delta(t) = 1}$$

$$\int_{1}^{2}\delta(t)\,dt = 0$$

calculate following → (i) $\int_{-\infty}^{\infty} \cancel{\delta(t} - \sin(t-\pi)\,\delta(t-\pi/2)\,dt$

$= \sin(\pi/2-\pi) = \sin(-\pi/2)$

$= -\sin\pi/2 = -1$

Q $\int_{-\infty}^{\infty}\sin(t-\pi)\,\delta(3t-\pi/2)\,dt = \frac{1}{3}\sin(\pi/6-\pi)$

$= +\frac{1}{3}\sin(-\pi/6)$

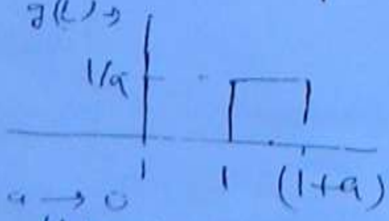
$= -1/6$

Q-(i) Find the value of the following integral.

(i) $\int_{-\infty}^{\infty}\frac{\sin(\pi t-\pi/2)}{(t^2+4)}\,g(t)\,dt$ where $g(t)$ is following pulse -

(ii)

$= \frac{\sin\pi/2}{5} = 1/5$



as $a \to 0$

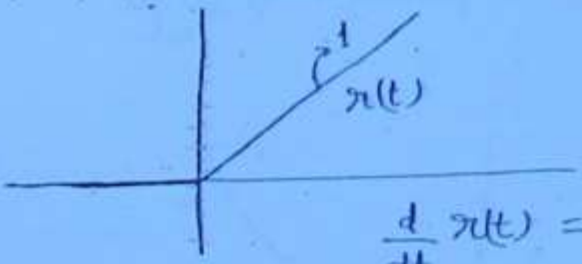
as $a \to 0$

$= (\cancel{1+a/2})\ \delta(t-1)$

15th oct 10

differentiation →

$f(t)$, $\frac{df(t)}{dt}$

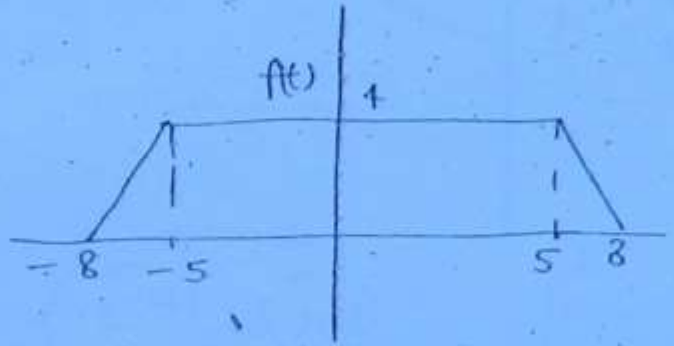


$f(t) = mt + c$

$\frac{df(t)}{dt} = m$

$\frac{d}{dt} r(t) = 1 \quad 0 \leq t \leq 1 \quad = u(t).$

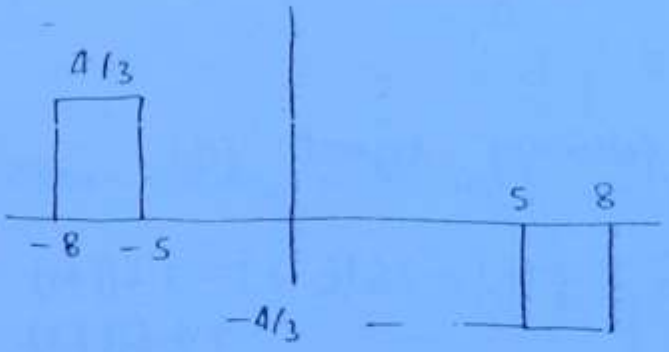
$\qquad\qquad 0 \quad$ otherwise



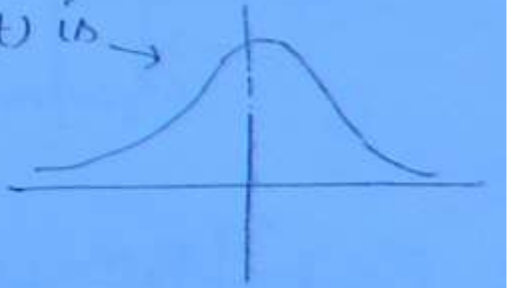
ar.K $f(t) = 4/3\, r(t+8) - 4/3\, r($
$- 4/3\, r(t-5) + 4$

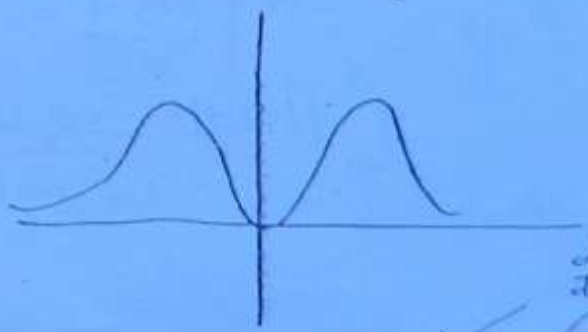
$\frac{d}{dt} f(t) = 4/3 - 4/3 - 4/3 + 4/$

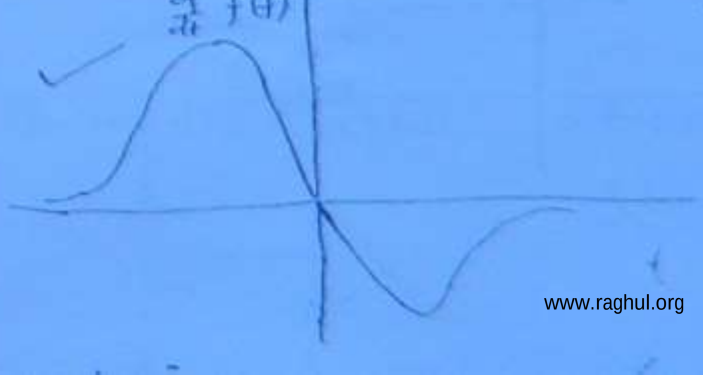
$= 0$ for $t \geq 8$



Q derivative of the following signal f(t) is →







area

$\int_{-\infty}^{\infty} f(t)\,dt = K$ (scaler → may be finite or infinite)

$\int_{-\infty}^{t} f(t)\,dt = g(t)$ area as a function of time

$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

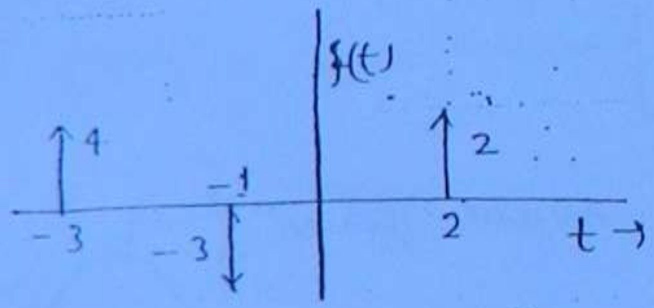
$$\int_{-\infty}^{t} \delta(\tau)\,dt = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

$= u(t)$

$\frac{d}{dt} u(t) = \delta(t)$

$r(t) = \int_{-\infty}^{t} u(t)\,dt$

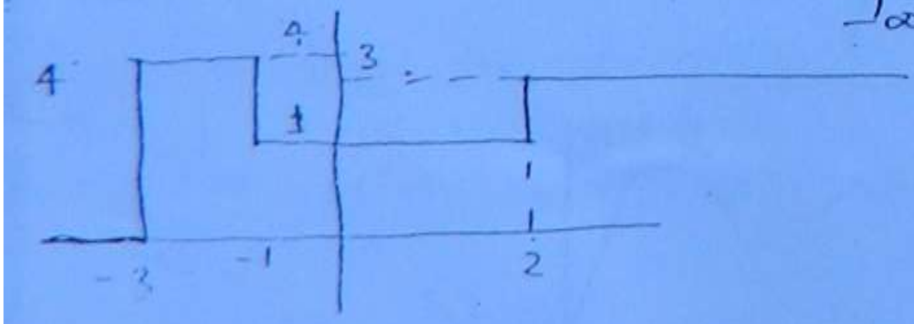
Calculate the integral of following signal f(t).



$f(t) = 2\delta(t-2) - 3\delta(t+1) + 4\delta(t+3)$

$\int_{-\infty}^{\infty} f(t) = 3$

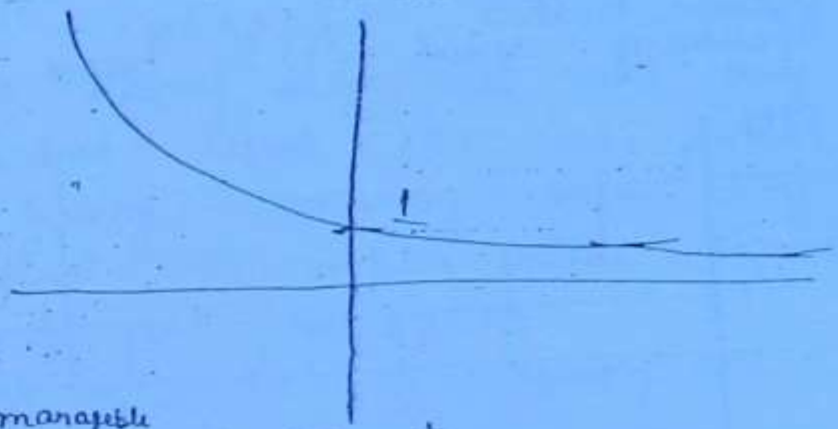
$\int_{-\infty}^{t} f(\tau)\,dt = 2u(t-2) - 3u(t+1) + 4u(t+3)$



$\frac{d}{dt} e^{-at} = -a e^{-at}$, $\frac{d}{dt} e^{at} = a e^{at}$

$$\int_{\infty}^{t} e^{-a\tau} d\tau = \left[\frac{e^{-a\tau}}{-a}\right]_{-\infty}^{t} = \frac{1}{a}\left[e^{-at} - \infty\right]$$

= not defined
unmanagable.



$$\int_{-\infty}^{t} \overbrace{e^{-at}\, u(t)}^{\text{managable}} dt = \int_{0}^{t} e^{-at} dt$$

$$= -\frac{1}{a}\left[e^{-at} - 1\right]$$

area under $e^{-at} u(t) = \int_{-\infty}^{\infty} e^{-at}\, u(t)\, dt$

$$= \int_{0}^{\infty} e^{-at}\, u(t) dt$$

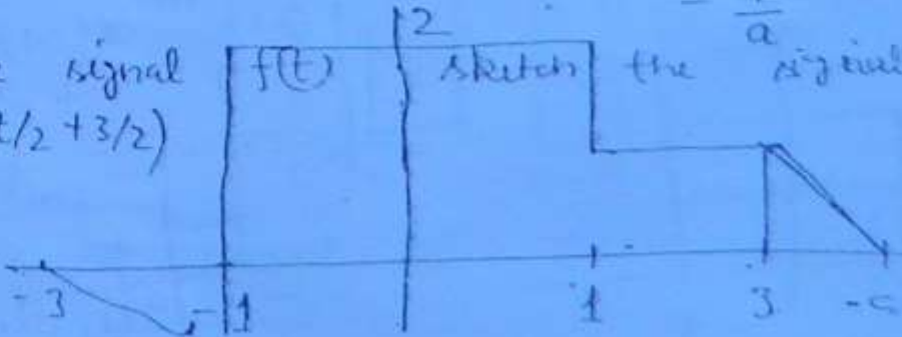
$$= +1/a$$

$$\int_{-\infty}^{t} e^{at} dt = \left[\frac{e^{at}}{a}\right]_{-\infty}^{t} = \frac{1}{a}\left[e^{at}\right]$$ unmanagable

$$\int_{-\infty}^{t} \overbrace{e^{at}\, u(-t)}^{\text{managable}} dt = \int_{-\infty}^{0} e^{at} dt = \frac{1}{a}\left[e^{at}\right]_{-\infty}^{0}$$

$$= \frac{1}{a}$$

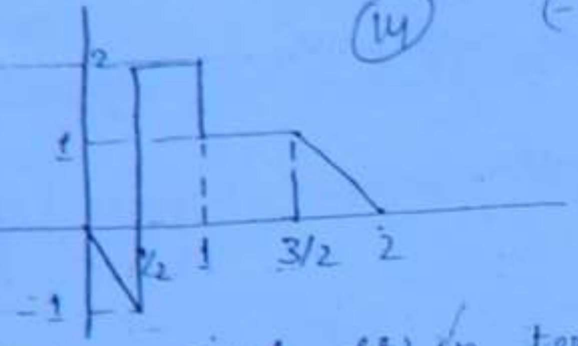
for above signal f(t) sketch the signal
$f(-t/2 + 3/2)$



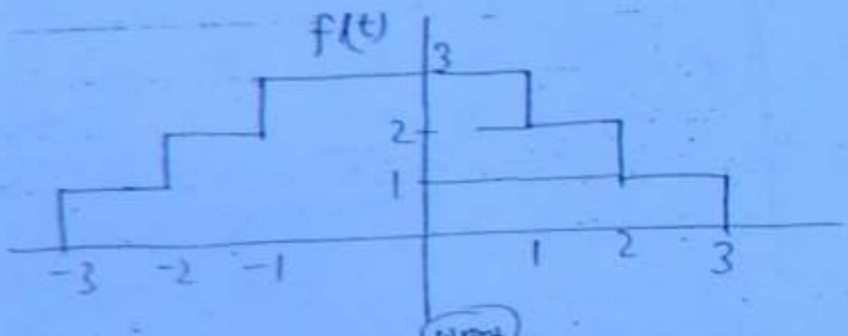
graph of f(-2t+3)

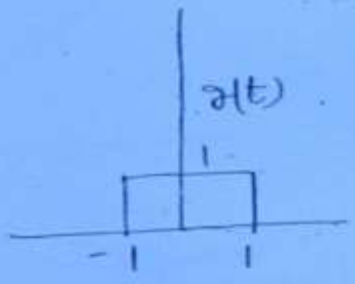
$(-2t+3) = t_1$

$t = -t_1/2 + 3/2$



Q. Represent the following signal f(t) in terms of g(t)





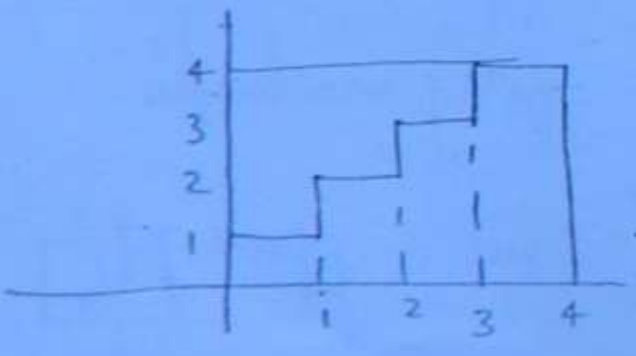
wrong

$$f(t) = \{g[t+2] + g(t+1) + g(t) + g(t-2) - g(t-3) - g(t-4)\}$$

$$f(t) = g(t/3) + g(t/2) + g(t)$$

Q. represent the following signal f(t) in terms of g(t)

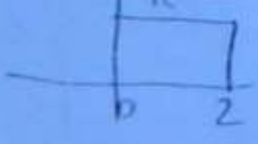
g(t) is same in previous question

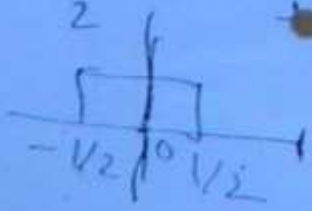


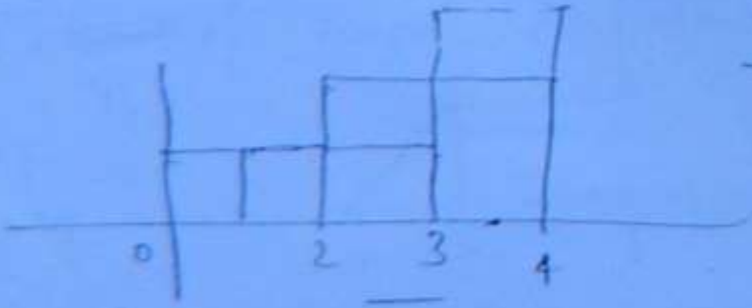
$$g(t-1) + g(t-2) + 2g(t-3) + ~~2g(t-4)~~ + 2g[2(t-7/2)]$$

Ans

$$f(t) = g(t-1) + g(t-2) + 2g(t-3) + 2g[2t-7]]$$







Types of signal →

can be real value signal or complex value signal

$f(t) = at$ real valued

$f(t) = at + ibt$ complex valued

$e^{jt} = \cos t + j\sin t$ Euler's identity

* Signals having only real value they are called as real valued signal $f(t) = \cos t, \sin t$, real valued signal

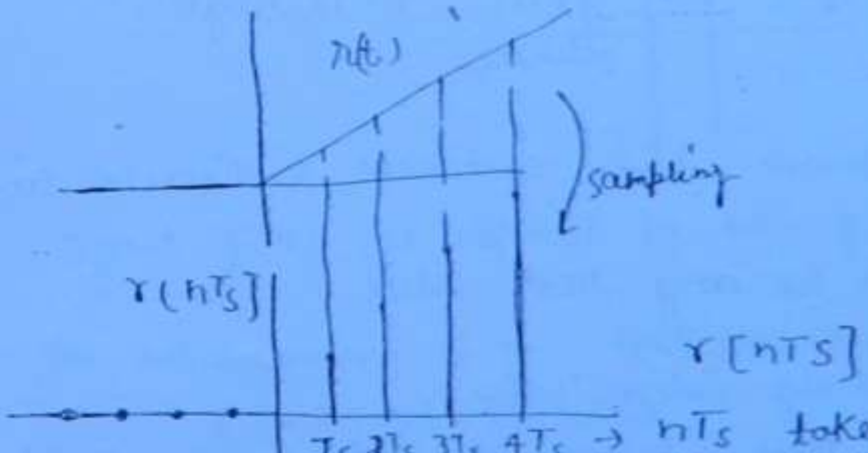
* Signals having imaginary value along with real value are defined as complex valued signal.

eg. $e^{jt} = \cos t + j\sin t$

real part $= \cos t$, imaginary part $= \sin t$

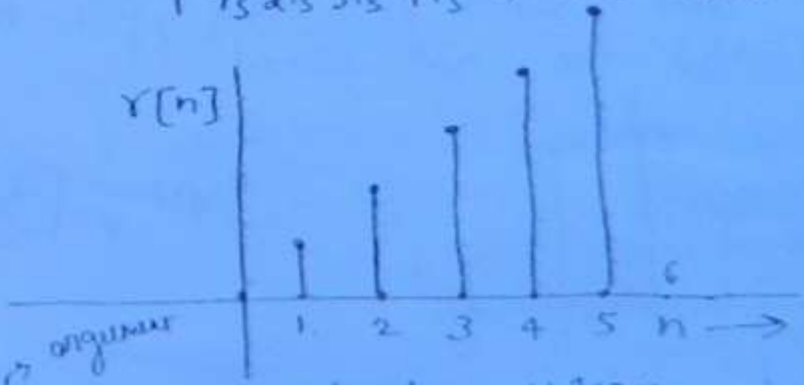
Continuous time & discrete time signal →

⊛ Continuous time signals are those signal having def value for every real value of time.



discrete time signals are those signals having defined values at some fixed insta and at other instant undefined.

take $T_s = 1$



$f[n] \rightarrow n$ must be integer , $f(t)$ ↳ any real valu

discrete time signal

$$f[n] = \{-1, \underset{\uparrow}{\overset{n=0}{1}}, 2, -2\}$$

$-1 \Rightarrow n=-1$

$1 \quad n=0$

$2 \quad n=1$

$-2 \quad n=2$

0 ~~otherwise~~ other values of n.

* Continuous time signal is that signal which is defined for all values of time.

* A discrete time signal is a signal which is defined only for specific values of time. It is not defined for other values of time. a discrete time signal is derived from a continuous time signal by a procedure called as uniform sampling and then selecting uniform sampling interval value to be 1.

$$\underset{\text{continuous}}{f(t)} \rightarrow f[nT_s] \rightarrow T_s=1 \rightarrow \underset{\text{discrete}}{f(n)} \quad n \text{ is integer.}$$

Fundamental difference b/w continuous & discrete time signal is $f(t)$

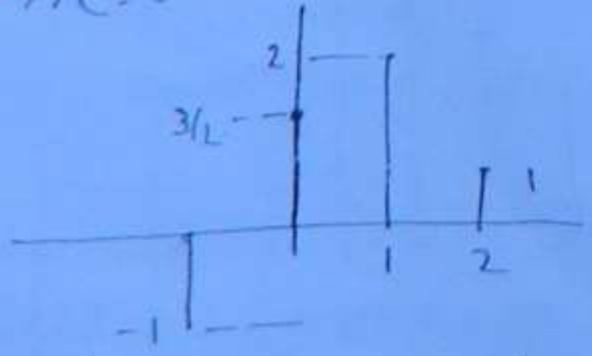
↳ t can be any real value

$f(n)$

↳ n → only integer value.

* For a discrete time signal f(n), value like f(4/3), f(5/6), & f(-2/3) are not defined.

an eg →

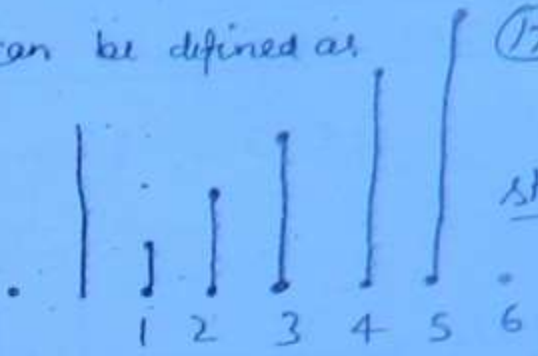


$$f(n) = \{-1, \overset{n=0}{3/2}, 2, 1\}$$

discrete time ramp signal can be defined as

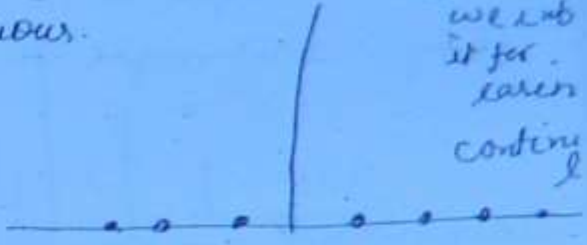
$r(n) = n \quad n \geq 0$

$\quad\quad\; 0 \quad$ otherwise



strictly

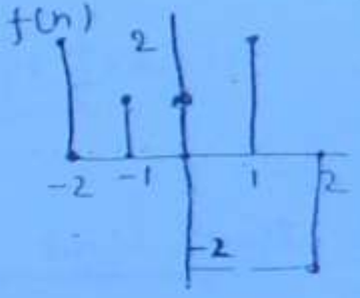
- n-axis discrete
  amplitude axis is continuous.



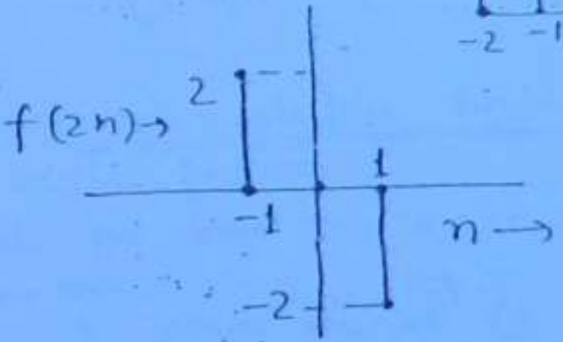
but we use it for easier conting

$f(n) \xrightarrow{n \to n-3} f(n-3)$

$f(2n) \longrightarrow$



[illegible]



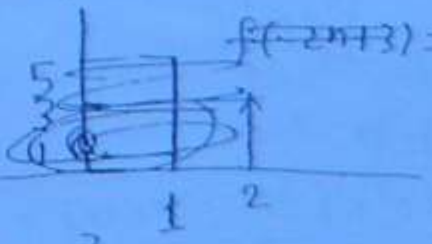
Q. Signal $f(n)$ is defined to be $f(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

for what values of n will the signal

$f[-2n+3]$ be zero

$f[-2(n-3/2)]$

- at $n = 0, -2, -4, \ldots, +2, 4 \ldots$
- $f[-2n+3] =$



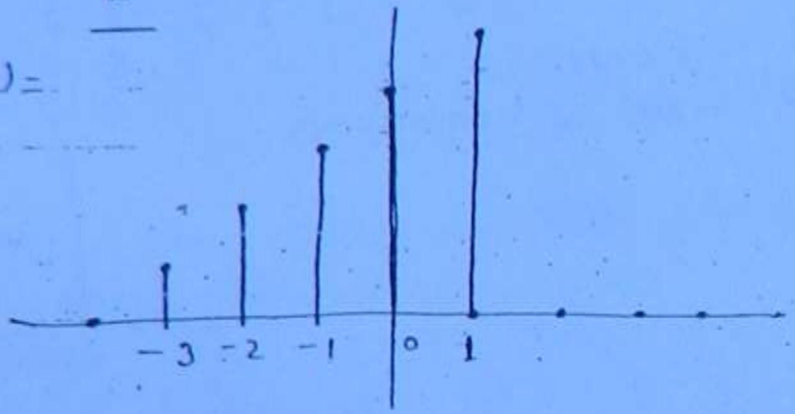
~~$f(-2n+3) = \{1, 5, 3\}$~~
$\uparrow$
0

$f(-2n+3) = \{\underset{\uparrow}{0}, 5, 3, 1\}$ 0 for $n \leq 0$ and 0 $n > 3$

discrete time unit step signal :→

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

f(n) =



$$f(n) = u[n+3] + u[n+2] + u[n+1] + u[n] + u[n-1] - 5\,u[n-2].$$

discrete time Unit impulse function →

area of $\delta(t) = 1$

$\delta[n] = 1 \quad n = 0$ → ~~real~~ by defining value 1 we are making it manageble.

$ 0 \quad n \neq 0$

17th oct 10 :

$$\delta[n] = \overset{\frown}{u[n]} - u[n-1]$$

$$\delta(t) = \frac{du(t)}{dt}$$

$$u[n] = \sum_{0}^{\infty} \delta[n-K]$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\,dt$$

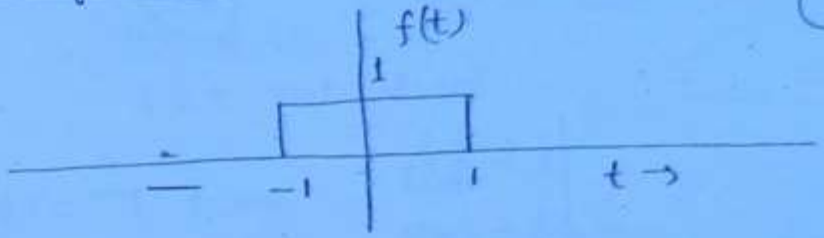
$$u[n] = \sum_{K=-\infty}^{n} \delta(K]$$

~~r[n]~~ =

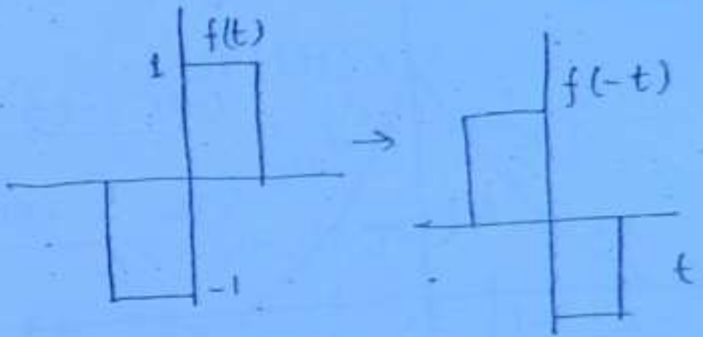
$$\boxed{u[n] = r[n] - r[n-1]}$$

$$r(n) = \int_{-\infty} \sum_{k=-\infty}^{n} u[k]$$

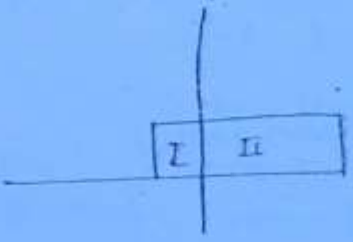
Even signal or odd signals :



$f(-t) = f(t)$ even signal .
or symmetrical signals



$f(-t) = -f(t)$
odd signal
or antisymmetri
signal .

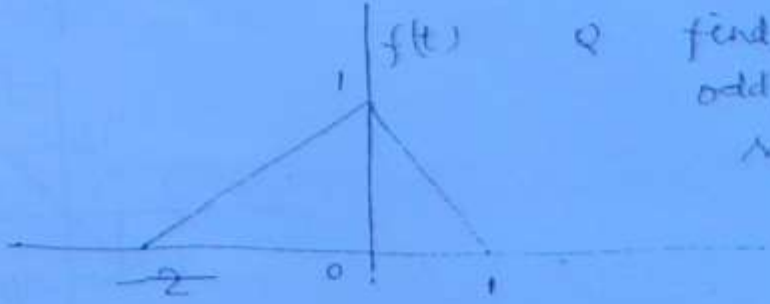


neither even nor odd signal

$f(t) + f(-t) \longrightarrow$ even signal (always)
$f(t) - f(-t) \longrightarrow$ odd signal

$f(t) = \frac{1}{2}[$ even signal + odd signal $]$
even part of $f(t)$ odd part of $f(t)$.

$\cos(t) = \cos(-t) \longrightarrow$ even signal or symmetrical signal
$\sin(t) = -\sin(-t) \longrightarrow$ odd signal or antisymmetrical sign
(opposite symmet

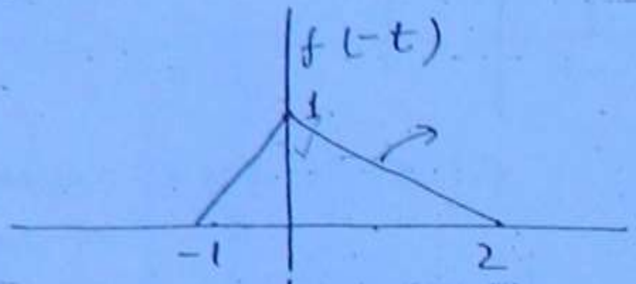


Q find the even and odd part of following signal f(t).

10

even part = $\frac{1}{2}[f(t) + f(-t)]$

=



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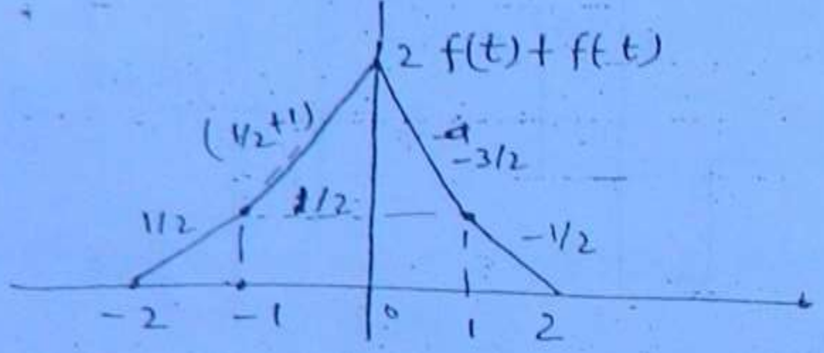
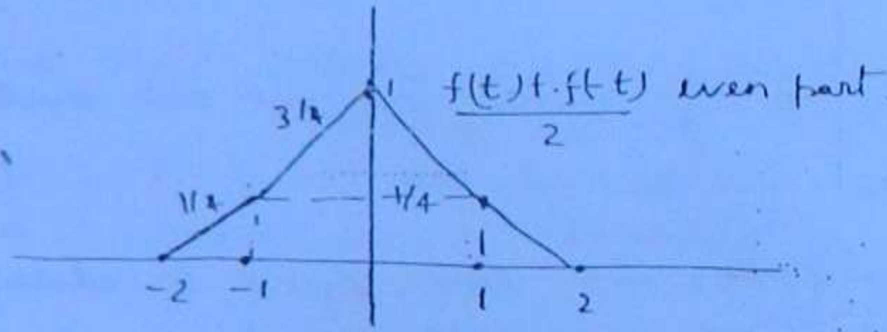
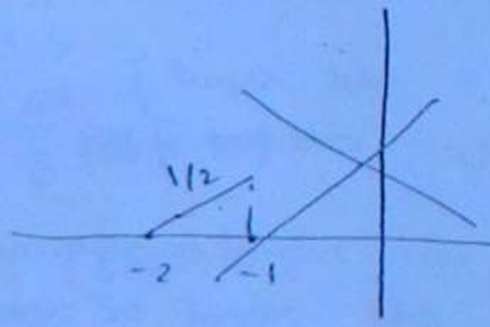
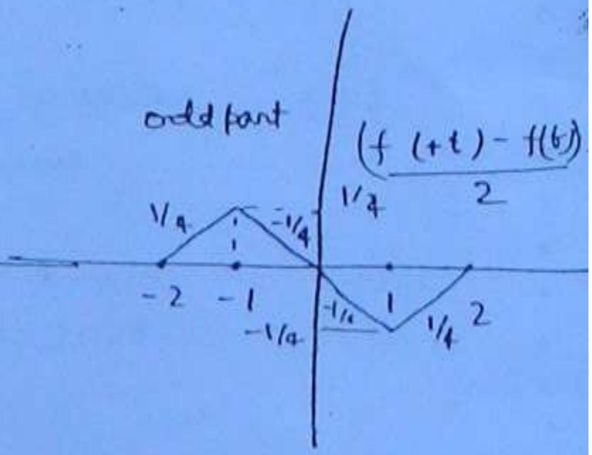
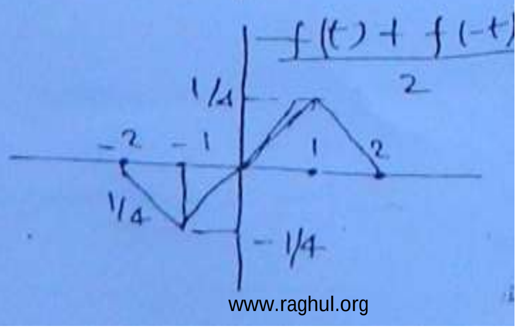




odd part





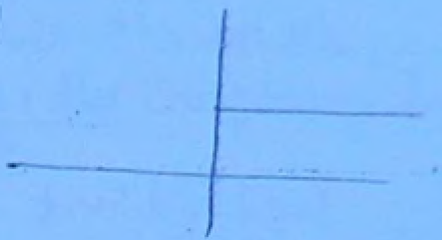


$\frac{1}{2}[u(t) + u(t)]$

$u(t) = 1 \quad t > 0$

$0 \quad t < 0$

$t = 0$ not defined

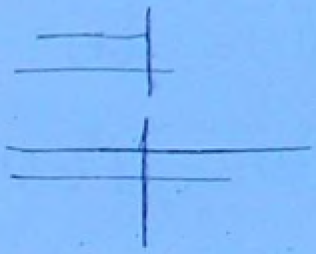


$u(t) = 1 \quad t > 0$

$0 \quad t < 0$

$1/2 \quad t = 0$

take any definition.



$$u_e(t) = \frac{u(t) + u(t)}{2} = 1/2$$

$$u_o(t) = \frac{u(t) - u(t)}{2} = \frac{1}{2}\operatorname{sgn}(t)$$

$\operatorname{sgn}(t) = 1 \quad t > 0$

$-1 \quad t < 0$

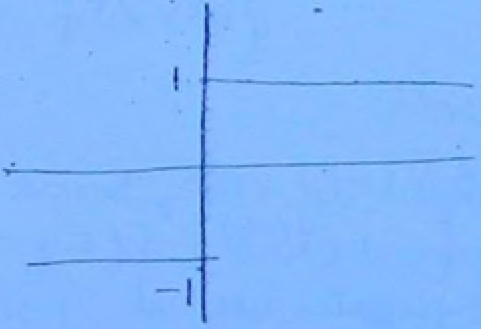
four definition any we can take

$t = 0$ undefined

or 1

or -1

or $= 0$



Q. find a even and odd part of $f(t) = \sin(t)\, u(t)$

$f(t) = e^{jt} = \cos t + j\sin t$

$f_e(t) = \frac{e^{jt} + e^{-jt}}{2} = \cos t$

$f_o(t) = j\sin t$

$f(t), f^*(t)$

(*) $f(t) = f^*(-t)$ even conjugate signal

conjugate symmetric signal

$f(t) = -f^*(-t)$ odd conjugate signal

or conjugate antisymmetric signal

$e^{jt} \rightarrow$ even conjugate signal

$f(t) = f^*(-t)$

$= [e^{j(-t)}]^* = e^{(-j)(-t)} = e^{jt}$

$f(t)$ je^{jt}

$-f^*(-t)$

$-[je^{j(-t)}]^* = -[-je^{jt}] = je^{jt}$

odd conjugate signal

$f(t) = e^{jt} = \underbrace{\cos t}_{\text{even}} + j\underbrace{\sin t}_{\text{odd}}$

* for an even conjugate signal

Real part $[f(t)]$ = even part of $f(t)$ → even in nature

odd part $[f(t)]$ = ~~odd~~ imaginary part of $f(t)$

$je^{jt} = \underbrace{-\sin t}_{\text{odd}} + j\underbrace{\cos t}_{\text{even}}$

(*) for a complex valued signal which is even conjugate in nature real part is always even and imaginary part is always odd.

(*) For a complex valued signal which is odd conjugate in nature real part is always odd and imaginary part is always even.

$f(t) = f_{oc}(t) + f_{oc}(t)$.

(23)

$f^*(t) = f^*_{ec}(t) + f^*_{oc}(t)$

$f^*(-t) = f^*_{ec}(t) + f^*_{oc}(-t)$

$= f_{ec}(t) - f_{oc}(t)$

J

o

$\sin t + j$

$\sin(-t) + j$

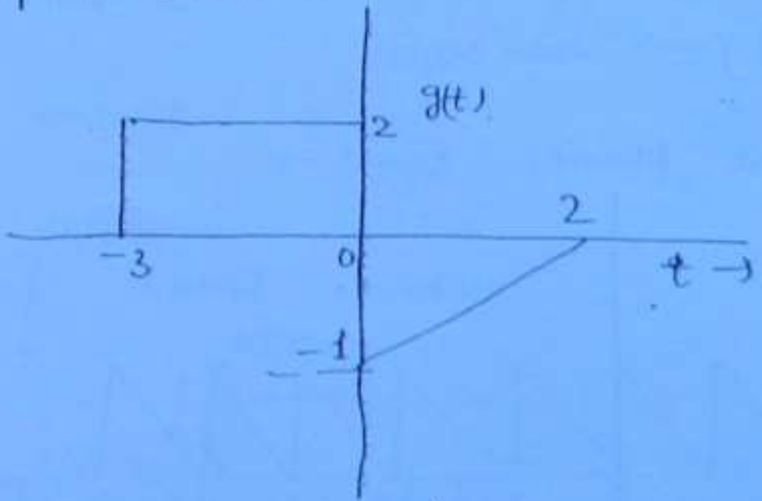
$\sin(-t) - j$

$-\sin t - j$

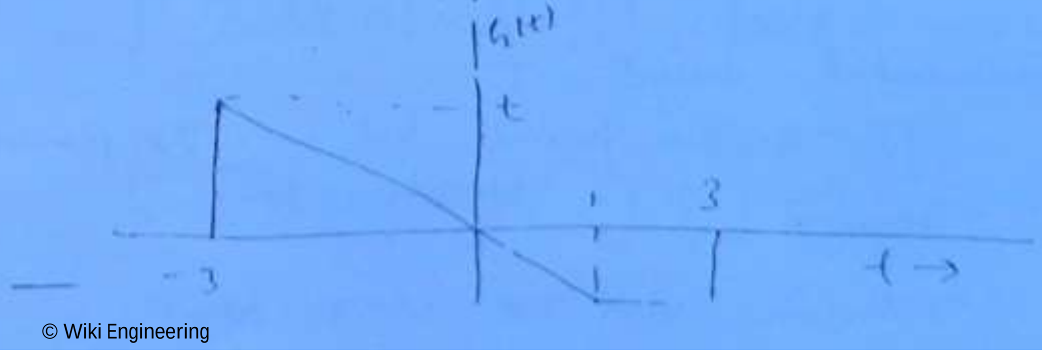
$f_{ec}(t) = \frac{1}{2}[f(t) + f^*(-t)]$

$f_{od}(t) = \frac{1}{2}[f(t) - f^*(-t)]$

Q. A complex valued signal f(t) is defined with a real part → [g(t) - g(-t)] and imaginary part which is {h(t) + {h(t)}} where g(t) & h(t) are as defined below.



then the signal f(t) is
even conj
✓ odd conju
neither even nor odd conj
can't comment



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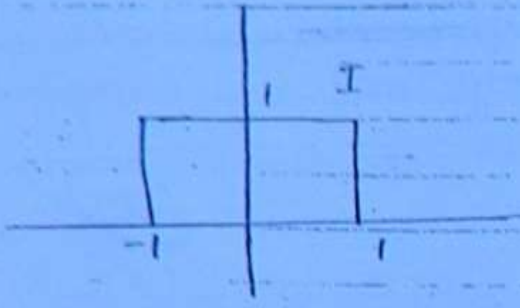
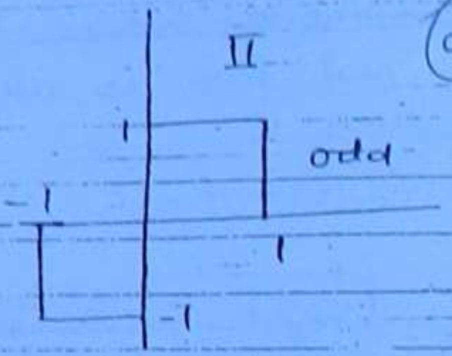
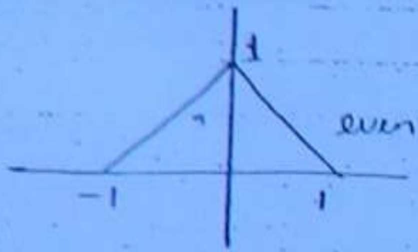
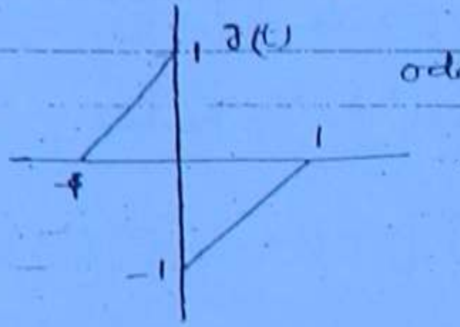
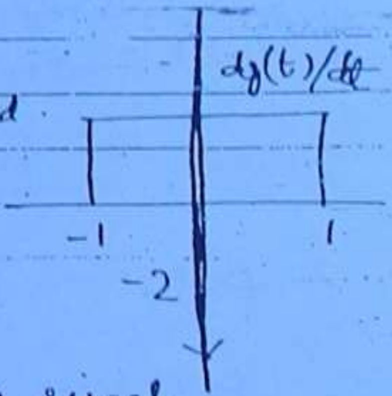










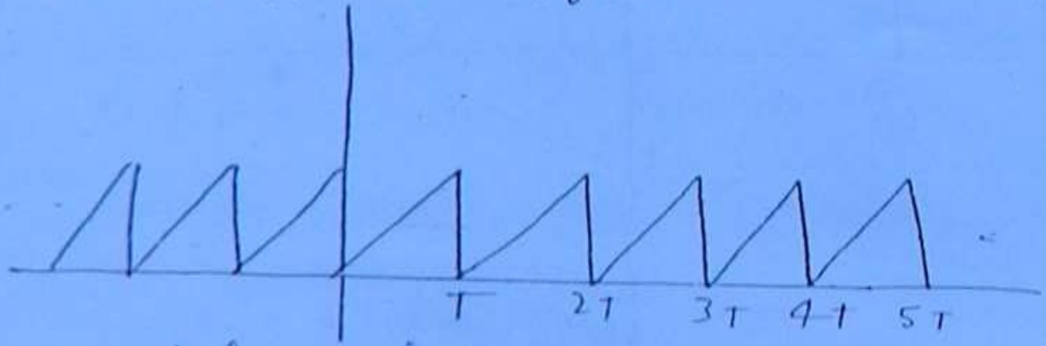
$$\int_{-a}^{a} f(t)\cdot dt = 2\int_{0}^{a} f(t)\,dt$$ for even signal

$$\int_{-a}^{a} f(t)\,dt = 0$$ for odd signal

$\frac{d}{dt}$ [even signal] = odd signal

$\frac{d}{dt}$ [odd signal] = even signal

eriodic and a.periodic signal →.



fundamental period = T

kT → time period but not the fundamental period.
↓
integer

any shifting will not change period.

$$f(t) = f(t \pm \underset{\text{integer}}{K}\overset{\text{fundamental period}}{T})$$

fundamental period of $\underset{K\ \text{integer}}{\sin Kt} \rightarrow \overset{\text{fundamental}}{\dfrac{2\pi}{|K|}} \rightarrow \overset{\text{also a peri}}{\dfrac{2\pi}{|K|} \times m}$ m

$\downarrow$

Can be 2π.

$\left.\begin{matrix} \sin t \\ \sin 2t \\ \sin 3t \\ \vdots \\ \sin Kt \end{matrix}\right\} \rightarrow$ all has period 2π (but this is not fundamental period for all except $\sin t$

$$\int_0^{2\pi} \underset{\substack{\uparrow \\ K \text{ is integer.}}}{\sin Kt}\, dt = 0. \quad \left(K \text{ complete cycle of period } \frac{2\pi}{|K|} \right)$$

so area = 0

$\sin \omega_0 t \longrightarrow \dfrac{2\pi}{\omega_0}$ fundamental period)

$\sin 2\omega_0 t \longrightarrow \dfrac{2\pi}{2\omega_0} \rightarrow \pi/\omega_0$

$\sin 3\omega_0 t \longrightarrow \dfrac{2\pi}{3\omega_0}$

common period of $\dfrac{2\pi}{\omega_0}$

$$\int_0^{2\pi/\omega_0} \sin \omega_0 t \cdot \sin 2\omega_0 t\, dt$$

$$= \frac{1}{2}\int_0^{2\pi/\omega_0} (\cos \omega_0 t - \cos 3\omega_0 t)\, dt = 0$$

$$\int_0^{2\pi/\omega_0} \cos\omega_0 t \cos 2\omega_0 t = \frac{1}{2}\int_0^{2\pi/\omega_0} (\cos 3\omega_0 t + \cos \omega_0 t)\, dt$$

$$= 0$$

$$\int_0^{2\pi/\omega_0} \sin\omega_0 t \cos 2\omega_0 t\, dt = \frac{1}{2}\int_0^{2\pi/\omega_0} [\sin 3\omega_0 t - \sin \omega_0 t]\, dt$$

$$= 0$$

B

$$\left.\begin{array}{l}\sin\omega_0 t\\ \cos\omega_0 t\end{array}\right\} 2\pi/\omega_0$$

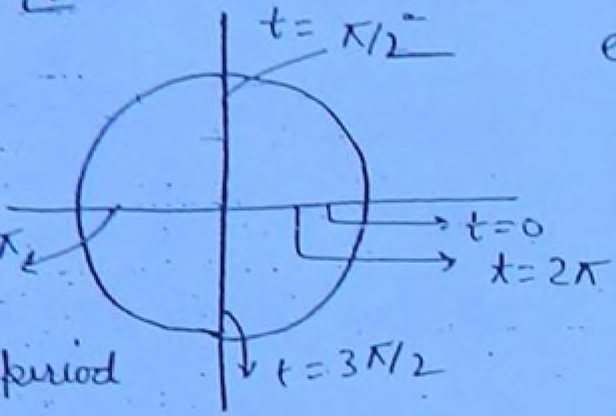
$$e^{j\theta} = \cos\theta + j\sin\theta = 1\ \angle\tan^{-1}(\tan\theta)$$

$$= \angle\theta$$

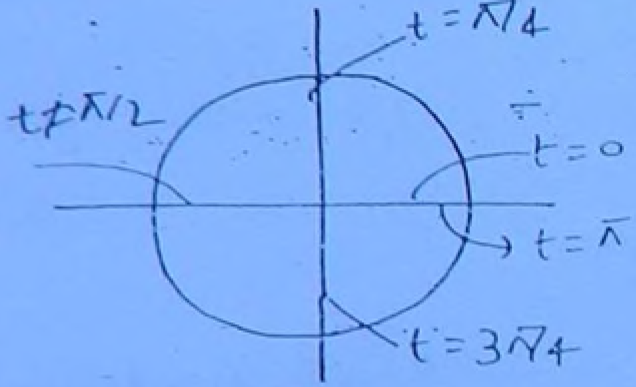
$$e^{jt} = 1\ \angle t$$

$$e^{jt} \xrightarrow{\text{prd}} 2\pi$$

phasor quantity covering the circumference of circle of unit radius and periodic with fundamental period 2π.



$$e^{j(2t)}$$
$$= 1\cdot\angle 2t$$



fundamental period $= \pi$

$e^{jKt} \rightarrow$ fundamental period $\dfrac{2\pi}{K}$.

$e^{j\omega_0 t} \rightarrow \dfrac{2\pi}{\omega_0}$

$e^{-j\omega_0 t} \rightarrow \dfrac{2\pi}{\omega_0}$ rotate in opposite direction.

Minimum No. of samples taken to repeat itself is defined as the fundamental time period of a discrete time periodic signal.

- No. of samples is always a integer and hence time period of a discrete time signal is always an integer.

* For a discrete time complex exponential $e^{j\omega_0 n}$ to be period the condition is ratio $\frac{2\pi}{\omega_0}$ must be rational, if it is rational the period

$$N = m \cdot \frac{2\pi}{\omega_0}$$

where m is selected to be a minimum possible integer such that above product is an integer.

$$e^{j\omega_0 n} \rightarrow \omega_0 \rightarrow (\omega_0 + 2\pi \underbrace{K}_{\text{integer}})$$

* A discrete time complex exponential signal there is no change in signal even if ω_0 is replaced by $(\omega_0 + 2\pi K)$ i.e. discrete time complex exponential signals the frequencies $\pi + 2\pi$, $\pi + 4\pi$, $\pi + 6\pi$ - - - so on, $\pi - 2\pi$, $\pi - 4\pi$ - - - so on, all denote the same discrete time complex exponential signal.

$$\text{same as} \quad \left.\begin{matrix} \cos\omega_0 n \\ \sin\omega_0 n \end{matrix}\right\} \rightarrow \overset{\text{rational}}{\boxed{\frac{2\pi}{\omega_0}}} \rightarrow \text{integer}$$

<u>Some of signals</u>

$$\underbrace{\underset{2\pi}{\overset{\sin t}{\downarrow}} + \underset{\pi}{\overset{\sin 2t}{\downarrow}}}_{\text{over all period} = 2\pi}$$

$$\frac{T_1}{T_2} = 2 \rightarrow \text{rational number}$$

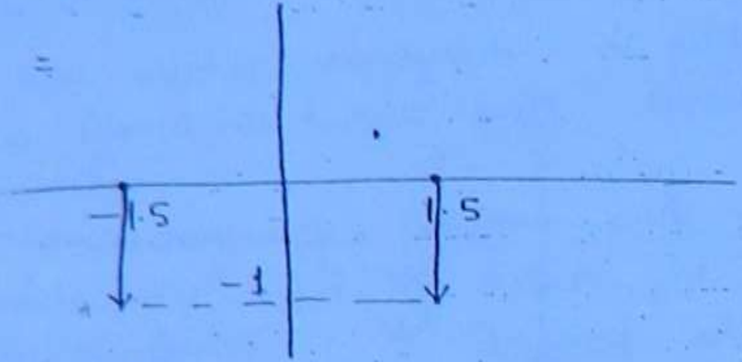
<u>$\sin t + \sin \pi t$</u> $\qquad \therefore \frac{T_1}{T_2} = \pi$ not rational

$T = 2\pi \qquad T = 2$

Repeat the above problem g(t) is defined as

$g(t) = f(t)\,[-\delta(t-1.5) + \delta(t+1.5)]$

the $g(t) =$



Q if the signal

$$f(t) = 3 + 4\sin(\pi/3\,t + \pi/2) + 6\cos(\pi/4\,t + \pi/3)$$

Power signal, calculate the power

$$= 9 + \frac{4^2}{2} + \frac{6^2}{2} = 35W$$

Q If signal $f(t) = e^{j2t}$ energy signal or power signal

power signal periodic signal

Power = 1

in calculation of power or energy we concentrate on amplitude not on phase.

in case of discrete time signals

$$E = \sum_{n=-\infty}^{\infty} |f[n]|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} |f[n]|^2$$

Find energy in the even conjugate part of the signal.

$$x[n] = \left\{ -4-j5 \quad \underset{\uparrow}{1+j2} \quad 4 \right\}$$

$$x_{ec}[n] = \frac{x[n] + x^*[-n]}{2}$$

$$= \left\{ -2.5j \quad \underset{\uparrow}{1} \quad 2.5j \right\}$$

$$\therefore \quad E_{ec} = 1 + (2.5)^2 + (2.5)^2$$

$$= 1 + 6.25 + 6.25$$

$$= 13.5 \text{ J}$$

$$\int_{-\infty}^{\infty} f^2(t)\, dt = \int_{-\infty}^{\infty} [f_e(t) + f_o(t)]^2 dt$$

$$= \int_{-\infty}^{\infty} f_e^2(t)\, dt + \int_{-\infty}^{\infty} f_o^2(t)\, dt + 2 \underset{\text{odd sign}}{\int_{-\infty}^{\infty} f_e(t)\, f_o(t)}$$

$$= E_e + E_o + 0$$

$$\boxed{E = E_e + E_o}$$

Causal or Non Causal signal·

↳ the signal which not start before zero

If signal start before zero then it is a non causal signal· (all periodic signals are non causal)

Deterministic or Random signal :

- * A signal which can be defined by well defined mathematical expression, it is called as deterministic signal·
- * A signal for which we can't give a well defined mathematical expression is defined as a random signal·

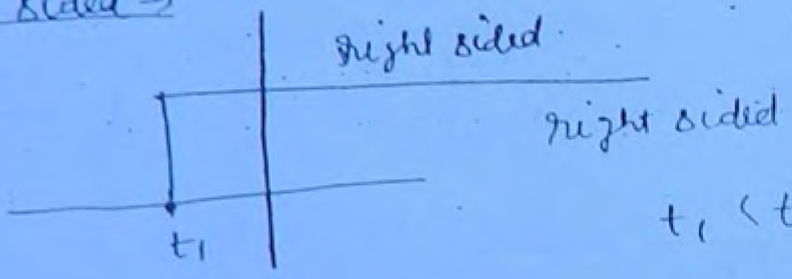
Bounded or unbounded signal →

* If the amplitude of signal have some finite boundaries for all values of time it is called as bounded signal

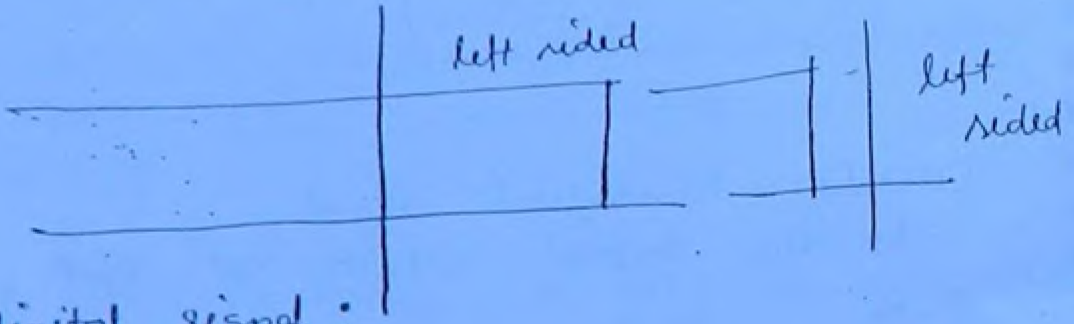
$$\left.\begin{array}{l}|f(t)| < \infty \\ |f(t)| < M \rightarrow \text{finite}\end{array}\right\} \text{for all } t,\ \text{bounded signal}$$

if signal value become infinite for any value of time, it is called as unbounded signal.

Right sided or Left sided →



$t_1 < t \leq \infty$



Analog or digital signal :

* A signal which can assume infinite no. of values for ~~its~~ amplitude is defined as analog signal.
* If a signal is allowed only to assume finite no. of amplitude then the corresponding signal is a digital signal. A digital signal is that signal which ~~disse~~ may discrete in both on time axis & amplitude axis.

$f(t)\ u(-t) = 0$

$f(t)$

Match the following: ③

List - I

expression of $f(t)$

A. $f(t)[1 - u(+t)] = 0$

B. $f(t) + K\frac{df(t)}{dt} = 0 \;\{(K = +ve)$

C. $f(t) + K\frac{d^2f(t)}{dt^2} = 0 \longrightarrow$

D. $f(t)[g(t) - g(0)] = 0$
arbitrary $g(t)$

List - II

nature of $f(t)$

(i) Increasing exponential

(ii) Causal signal

(iii) decreasing exponential

(iv) Sinusoidal

(v) Simpulse

$A \to f(t) = f(t)u(t)$

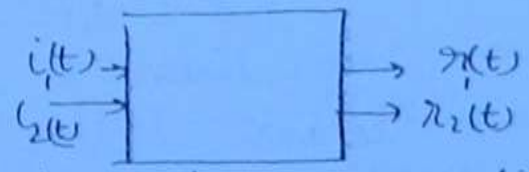
$t < 0$

$f(t) = 0$

$$\left\{ \begin{array}{l} C \to (iv) \\ B \to (iii), \\ A \to (ii) \\ D \to (v) \end{array} \right\}.$$

$f(t)\ g(t) = f(t)\ g(0)$

System :



⊛ A system is a quantity which maps a set of i/P signals to a set of a o/P signals

we can understand (or represent) a system by

(i) i/P - o/P relationship.

(ii) Physical composition → (NE, ECD, AE, DE)

(iii) differential equation or difference equations.

(iv) Unit impulse response $h(t)$, $h[n]$

(v). Transfer function $H(\omega)$, $H(S)$, $H(z)$

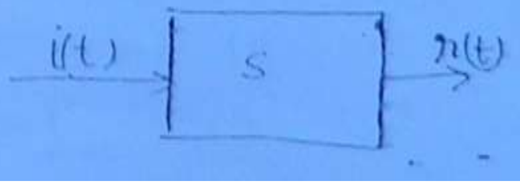
(VI) State variable

$V = L\frac{di}{dt}$

$i = C\frac{dv}{dt}$

our

time constant



$i(t) \xrightarrow{S} r(t)$

Linear system or Nonlinear system.

$i(t) \xrightarrow{S} r(t)$ where a is a real or imaginary quantity a constant [like 2 or 2j]

$a\, i(t) \xrightarrow{S} a\, r(t)$ homoginity principle

or

$i_1(t) \xrightarrow{S} r_1(t)$

$i_2(t) \xrightarrow{S} r_2(t)$

$\Big\{ \; i_1(t) + i_2(t) \xrightarrow{S} r_1(t) + r_2(t)$

superposition principle or additivity principle

$a\, i_1(t) + b\, i_2(t) \xrightarrow{S} a\, r_1(t) + b\, r_2(t)$

Linearity principle.

(*) A system satisfying both homoginity & superposition principle then it is said to linear system.
If it can not satisfy one of these principle or both, it is defined as non linear system.

$r(t) = 2i(t) + 3$ → not linear

$r(t) = \log i(t)$ → not linear

$r(t) = i^2(t)$ → non linear

$r(t) = t\, i(t)$ → linear

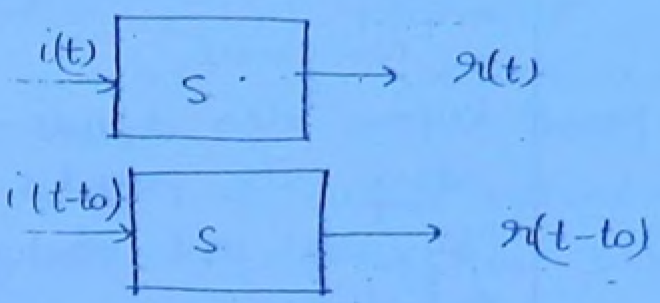
$r(t) = \sin t\, i(t)$ → linear

$r(t) = \int_{\infty}^{t} i(t) d\tau$ → linear /

$r(t) = \int_{-5}^{5} i(t) d\tau$ → linear

✓ $r(t)$ = real part of $\{ i(t) \}$ not linear.
it does not hold homoginity for an $a = jb$
[illegible] $a\, i(t)$ will not be $a\, r(t)$ ∴ non linear } real /

Time varient or time invarient :→



time invarient system.

* The system is defined as time invarient system if the response is delayed by the same amount as delay given to the system.

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* If response to $i(t-t_0)$ is not equal to $r(t-t_0)$, system is called as time varient system

$i(-t)$ $\qquad\qquad r(t-t_0) \rightarrow i(-t+t_0)$

$r_1[-t-t_0]$

$\downarrow$

$\rightarrow i'(t) \rightarrow r'(t) \rightarrow i'(-t)$.

$i(-t-t_0)$

$\downarrow$

$\neq$

$r(t-t_0)$

time varient system

$r(t) = 2\,i(t) + 3 \rightarrow$ time invarient

$r(t) = \log i(t) \rightarrow$ time invarient

$r(t) = i^2(t) \rightarrow$ T. I.

$r(t) = t\,i(t) \rightarrow$ T. V

$r(t) = \sin t \cdot i(t) \rightarrow$ T. V

$r(t) = \int_{-\infty}^{t} i(\tau)\,d\tau \rightarrow$ T. I

$r(t) = \int_{-5}^{5}$ [illegible]

T. V

$r(t) = i$ [illegible]

⊛ Causal system & Non Causal system →

Cause → Causal

Causal	Non causal
(i) response depends upon present & past i/ps	Response also depends on future along with present & past i/ps
(ii) Physically realizable	Physically not realizable.
(iii) Nonanticipatory	Anticipatory

* Non causal system also become physically realizable when the data which is being operated upon or when the i/p data is recorded data. But by default we consider data to be real time data and hence only causal systems are physically realizable systems.

$$i(t) \qquad i(t-t_0) \qquad i(t+t_0)$$

$$i=0 \quad i(0) \qquad i(-t_0) \qquad i(t_0)$$

$$r(t) = f[i(t), i(t-t_0)]$$

$$r(t) = f[i(t-t_0)]$$

$$t_0 \geq 0$$

* $r(t) = i(-t)$

non causal

$r(1) = i(-1)$

$r(-1) = i(1) \rightarrow$ depend future i/p

$r(t) = i(t) + i(t-2) + i(t-4) \rightarrow$ Causal system

$r(t) = i(2t) \rightarrow$ non causal system

$r(1/2) = i(1)$

$r(t) = i(1/2\, t)$ – non causal

$r(-1) = i(-1/2) \rightarrow$ future i/p

(*) $r(t) = i(at) \longrightarrow$ always non causal

$r(t) = i^2(t) \rightarrow$ causal system

$r(t) = \cancel{i^2(t)}\ i(t^2) \rightarrow$ non causal

$r(t) = i(\sin t) \rightarrow$ non causal

$r(\pi/2) = i(1)$

$r(-\pi/2) = i(-1) \longrightarrow$ future i/p

$r(\pi) = i(\sin\pi) = i(0)$

$r(-\pi) = i(0)$

* Systems can be static or dynamic

If there is no arrangement for memory in an electrical system, it is called as static system

If there is arrangement of memory in electrical system, it is called as dynamic system

$r(t) = i^2(t)$ – static system

or $\log i(t)$, $t\, i(t)$

$\sin t\ i(t)$

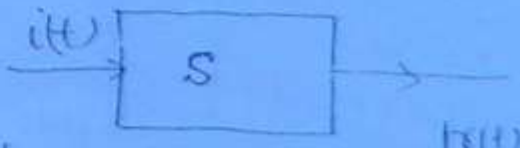
$2\, i(t) + 3$

$A \cdot i(t)$ } – Linear, static, time invarient)

* For a static system to be linear & time invarient only way the response can be related to i/p is $r(t) = A\, i(t)$

Stable or Unstable system :→

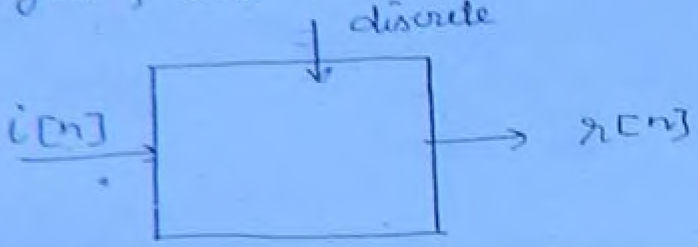
BIBO stability



$|i(t)| < M$

$|r(t)| \leq N$

* A system can become noninvertible in following cases
(i) if more than one i/p to system is generating the same response for the system.
(ii) if the response of the system to a nonzero i/p is zero; ~~and~~



* verify whether ~~is~~ ~~dos~~ following discrete time system is linear, time invarient, causal, static, stable, invertible.

$$r[n] = \sum_{k=-\infty}^{n} i[K] \rightarrow \text{accumulator}$$

↓
linear, time invarient, causal, dynamic, unstable, invertible

~~i[K-n0]~~

$$r'[n] = r[n] - r[n-1]$$
$$= i[n]$$

$i'[n] = i[n-n_0]$

$i'[K]$

$i[K-n_0]$

$i[n] = i[n-n_0]$

$\sum_{K=-\infty}^{n-n_0} i[K]$

$i[K-n_0]$

$\sum_{-\infty}^{n} i[K-n_0]$

$\sum_{-\infty}^{n-n_0} i(m)$

Q. Find weather system defined as
$$r[n] = i[n] - i[n-1]$$; check^n
for L, T, C, S, I.

this system → linear, time invarient, causal, stable

Invertible system because we can select a system having respon

$$r'[n] = \sum_{K=-\infty}^{n} r[K] = \sum_{K=-\infty}^{n} \{i[K] - i[K-1]\}$$

$\{\ldots i[n-2] + i[n-1] + i[n]\} = i[n]$

[illegible]

Q. If P → defines linearity
Q → defines Time invarient
R → Causality
S → Stability

A discrete time system defined by i/p o/p relationship.

$y[n] = x[n] \quad n>0$
$\quad\quad 0 \quad n=0$
$\quad\quad x[n+1] \quad n<0$

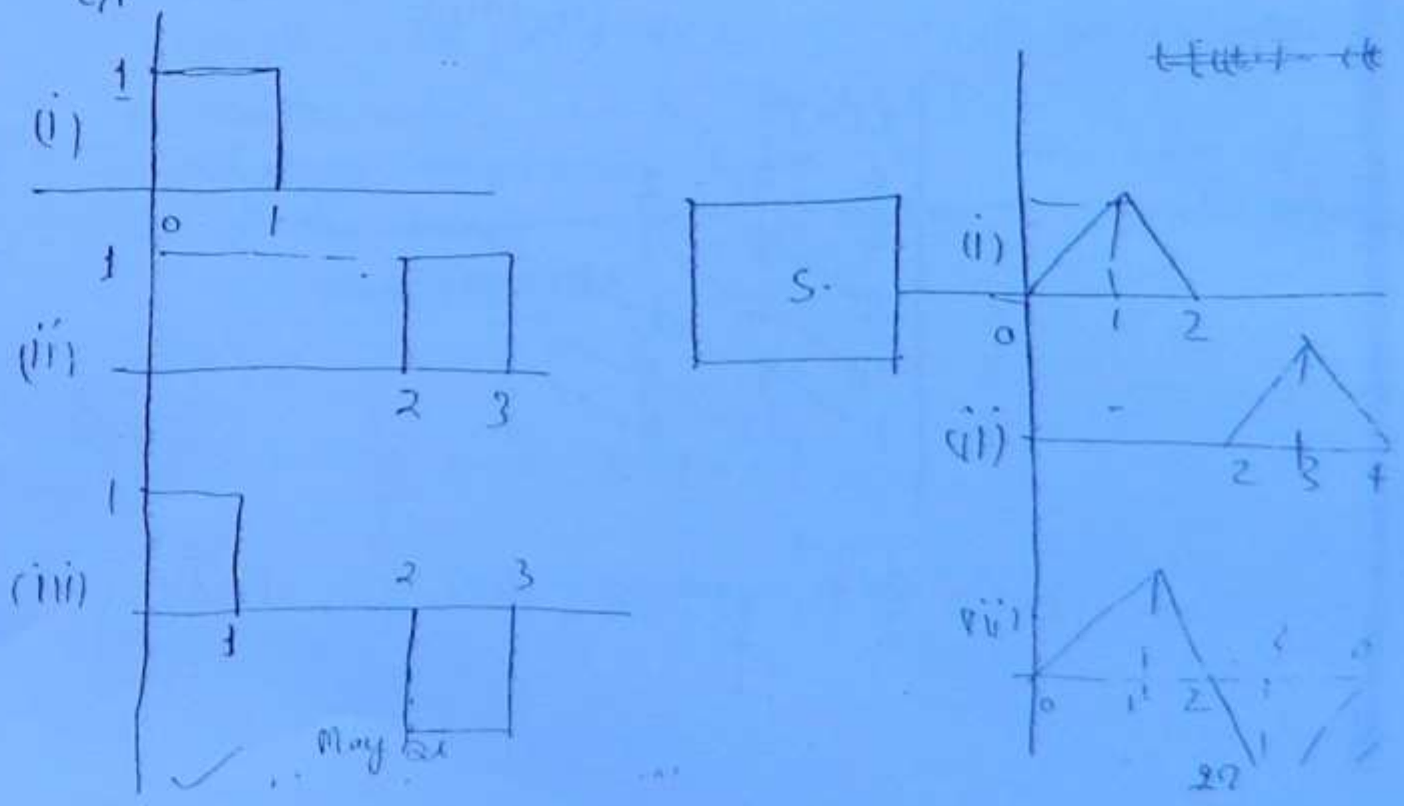
where x[n] and y[n], i/p and o/p of the system
system is

(i) P, Q, R, S
(ii) P, Q, S but Not R
✓(iii) P, S but Not, Q, R
(iv) P but no Q, R, S.

$y[n] = u[n-1]\,\underline{x[n]} + u[n+1]\,x[n+1]$

Linear, Not causal,
Stable, time varient

Q. *A system S has the following for the considered i/p



$\boxed{\text{Length of resultant signal } l \text{ always } = L_1+L_2-1}$

Lower end of resultant signal = $\underline{(n_1+n_2)}$

$n_1 \rightarrow$ Lower ento ~~at~~ end of $f[n]$
$n_2 \rightarrow$ Lower end of $h[n]$

When we convolve $f[n]$ & $h[n]$

A_1 summation of all sample of $f[n] = \sum_{K=-\infty}^{\infty} f[K]$

A_2 " " " of $h[n] = \sum_{K=-\infty}^{\infty} h[K]$

$$A_1 \cdot A_2 = \sum_{K=-\infty}^{\infty} y[K] \qquad y[n] = f[n] * h[n]$$

$$=$$

$n_2 \rightarrow$ upper end of $f[n]$
$n_3 \rightarrow$ " " " $h[n]$
upper end of $y[n] = f[n] \circledast h[n]$
$= (n_2+n_3)$

	h[n]			
	4	3	2	1
1	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

$$f[n] = \left\{ \overset{n=-1}{4}, \underset{\uparrow}{11}, 20, 30, 20, 11, \overset{n=5}{4} \right\}$$

* whenever two discrete time signals are convolve

(i) ~~Point~~ Resultant of convolution will have a length which is equal to sum of individual lengths of the signal being convolved −1 · (minus 1) ㊴

$$l = L_1 + L_2 - 1$$

(ii) Resultant of convolution will have extends which is equals to some of individual extends of the signal being convolved.

(iii) Some of the sample values is resultant of convolution value is same as the product of the some of the sample values of individuals signals being convolved

if $\boxed{h[n] = u[n]}$ not possible to follow this procedure.

⊛ The above method is suitable when the no of samples in the i/p signal and the impulse response are finite in no.

Q. Two discrete time signals $x[n]$ & $h[n]$ each of lengths 3 & 5 are convolved The maximum possible sample value of $x(n)$ is L , max^m possible value sample value is K for $h[n]$

What is maximum value of the some of all the sample value in resultant convolve signal

~~[illegible]~~

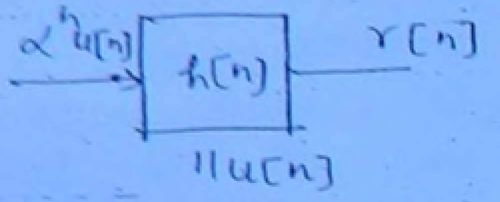
$$x[n] = \{L, L, L\}$$

$$h(n) = \{K, K, K, K, K\}$$

$$\left\{\sum_{n=-\infty}^{\infty} y[n]\right\}_{max}$$ ~~[illegible]~~ $= 3L \times 5K = 15LK$

$$\sum_{K=-\infty}^{\infty} r[K] = \sum_{K=-\infty}^{\infty} \overset{\text{impulse respon}}{h[K]} \cdot \sum_{K=-\infty}^{\infty} \overset{\text{i/p.}}{x[K]} \qquad (48)$$

$$\boxed{\sum_{K=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x[m]\, h[K-m] = \sum_{K=-\infty}^{\infty} h[K] \cdot \sum_{K=-\infty}^{\infty} x[K]}$$



$$r[n] = \alpha^n [u[n]] \otimes u[n]$$

$$r[n] = \sum_{K=-\infty}^{\infty} \alpha^K u[K]\, u[n-K]$$

$$= \left[\sum_{K=0}^{\infty} \alpha^K u[K]\, u[n-K] \right] \quad \text{~~u[n-K]~~}$$

$$r[n] = \left[u[n] + \alpha u[n-1] + \alpha^2 u[n-2] + \cdots \alpha^r u[n-r] + \cdots \right]$$

$$= 0 \qquad n < 0$$

$$r[n] = \left(\frac{\alpha^{n+1} - 1}{\alpha - 1} \right) \qquad n \geq 0$$

$n = 0 \quad 1$

$n = 1 \quad (1+\alpha)$

$n = 2 \quad 1+\alpha+\alpha^2$

$$n = r \rightarrow 1 + \alpha + \alpha^2 + \cdots \alpha^r$$

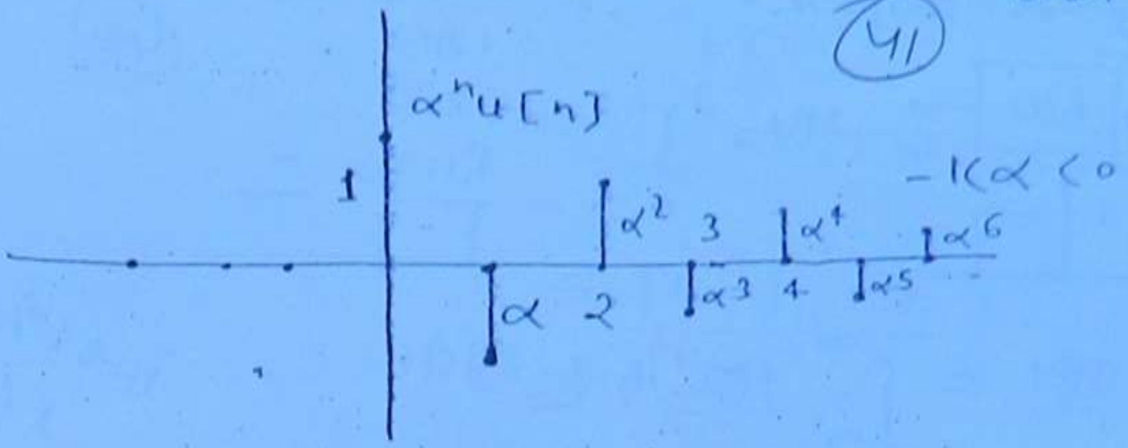
$$= 1 \frac{[\alpha^{r+1} - 1]}{\alpha - 1}$$

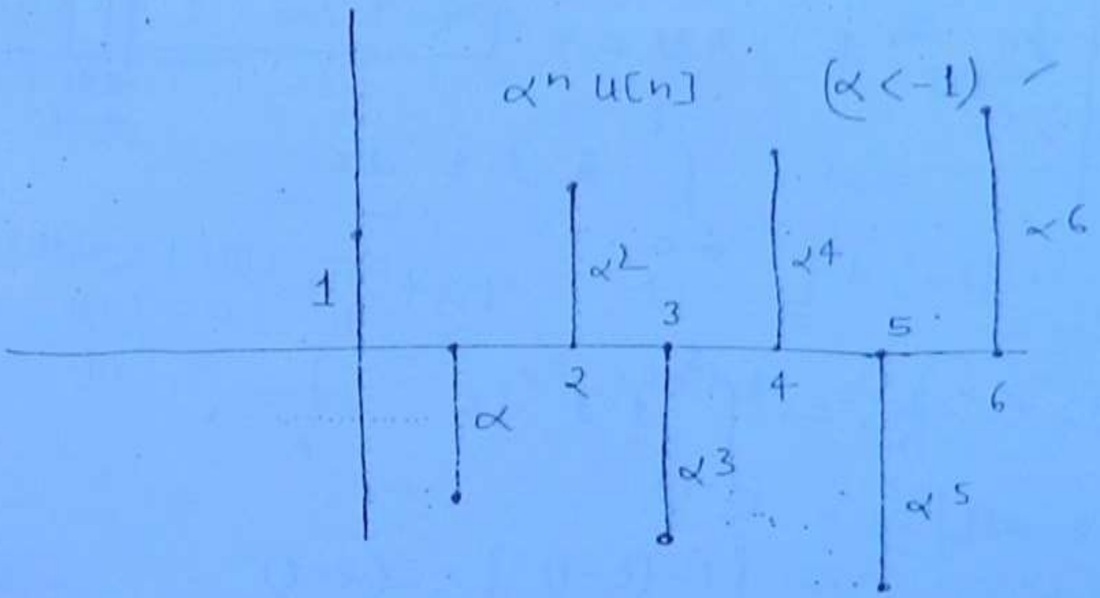
$$r[n] = \left(\frac{\alpha^{n+1} - 1}{\alpha - 1} \right) u[n$$

sketch the graphs $\alpha^n u[n]$ when $0<\alpha\le 1$

when $-1<\alpha<0$

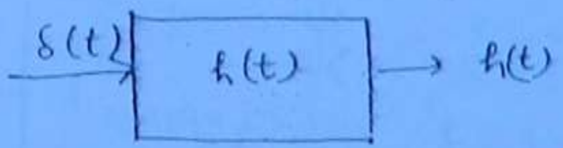
$\alpha<-1$





20th oct 10

$$f(t) = \int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,d\tau$$



for any i/p $f(t)$ response $r(t)$

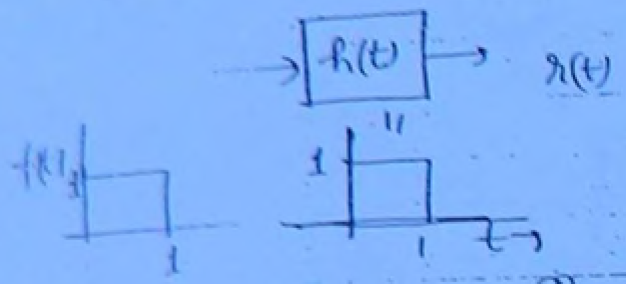
$$r(t) = \otimes \int_{-\infty}^{\infty} f(\tau)\,h(t-\tau)\,d\tau \longrightarrow \text{convolution integral}$$

$$\Downarrow$$

$$f(t) \otimes h(t)$$

Q. Find the response of a continuous time LTI system with impulse response:

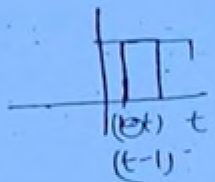
(42)



$$r(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau$$

h(-t+0)

h(-τ)



for $t < 0 \Rightarrow r(t) = 0$

for $t \geq 0$.

$$= \int_{0}^{t} 1 \cdot 1 \, d\tau$$

$$= t \qquad 1 \geq t \geq 0$$

for $2 \geq t > 1$ $= \int_{(t-1)}^{1} 1 \cdot 1 \, d\tau$

$$= [1-(t-1)] = (2-t)$$

for $t > 2$

$r(t) = 0$

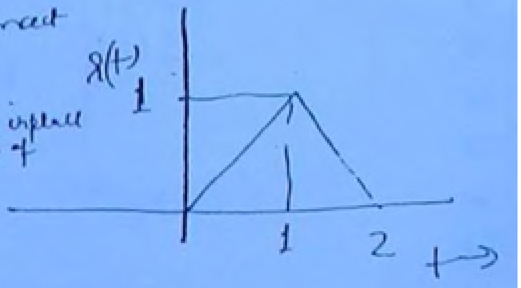
both correct

$r(t) =$ t, $1 \geq t \geq 0$ or > 0

$(2-t)$, $2 \geq t > 1$ or $t < 2$ replace $(t \leq 2)$

0, $t > 2$

0, $t < 0$



Repeat the above problem if the $f(t) = u(t)$ and the $h(t) = u(t)$

$r(t) = 0 \quad t < 0$

$r(t) = t \quad t \geq 0$ Ans

Q. Repeat the problem if $f(t) = u(t). e^{-t}$, $h(t) = u(t)$

(43)

$r(t) = 0 \quad t < 0$

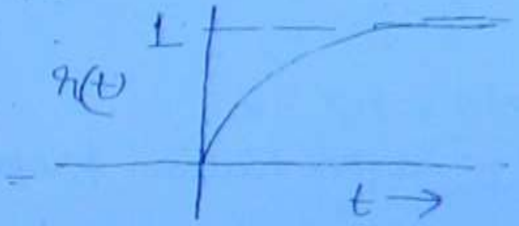
$r(t) = \int_0^t e^{-t}. 1 \, dt \quad \underline{-} \; t > 0$

$= -[e^{-t}]_0^t$

$r(t) = (1 - e^{-t}) \quad t > 0$

$0 \quad t = 0$

$\boxed{r(t) \to 1 \text{ as } t \to \infty}$



Q. $e^{-t} u(t) \otimes t\, u(t) \longrightarrow$

$\int_0^t -(e^{-t} \circ t)$

$t < 0 \quad r(t) = 0$

$t > 0 \quad r(t) = \int_0^t e^{-\tau} (-\tau + t) \, d\tau = -\int_0^t \tau e^{-\tau} d\tau + \int_0^t t e^{-\tau} d\tau$

$= -[(1 - e^{-t}) - t e^{-t}]$

$+ t [1 - e^{-t}]$

$r(t) = t e^{-t} + (t - 1)(1 - e^{-t})$

$t \geq 0$

Q. $e^{-t} u(t) \circledast e^{-t} u(t)$

$t < 0 \quad r(t) = 0$

$t > 0 \quad r(t) = \int_0^t e^{-\tau} e^{\tau - t} d\tau$

$e^{\tau} u(-\tau)$

$r(t) = t e^{-t} \quad t \geq 0$

$e^{(\tau - t)}$

Q. $e^{at} u(t) \otimes e^{-bt} u(t) =$

$t < 0 \quad r(t) = 0$

for $t > 0 \quad = \int_0^t e^{a\tau} e^{b(\tau - t)} d\tau = e^{-bt} \int_0^t e^{(a+b)\tau} d\tau$

$t \geq 0 \quad r(t) = \frac{e^{at} - e^{-bt}}{a+b} \qquad r(t) = \frac{e^{-bt}}{a+b} [e^{(a+b)t} - 1]$

⊛ $r(t) = f(t) \otimes h(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau$

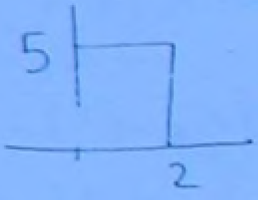
$$\frac{d}{dt} r = \frac{d}{dt}\left[f(t) \otimes h(t) \right] = \int_{-\infty}^{\infty} f(\tau) \frac{d}{dt} h(t-\tau)\, d\tau$$

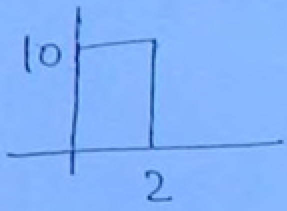
$$= \int_{-\infty}^{\infty} f(\tau)\, h'(t-\tau)\, d\tau$$

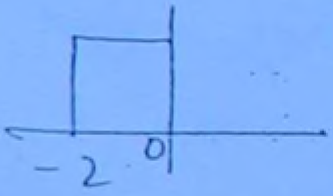
$$= f(t) ⊛ h'(t)$$

differentiation property $\boxed{\frac{dr}{dt} = f(t) \otimes \frac{dh}{dt}}$

Q Calculate the convolution of following two pulses







$r(t) = 0 \quad t < 0$

$2 \geq t > 0$

$$r(t) = \int_0^t 5 \cdot \times 10\, dt = 50t$$

$t = 2 \quad r(t) = 100$

for $4 > t > 2$

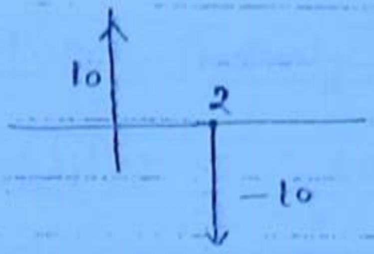
$$\int_{t-2}^{2} 5 \times 10\, dt$$

$$= 50[4-t]$$

$4 \geq t > 2$

$0 \quad t \geq 4$

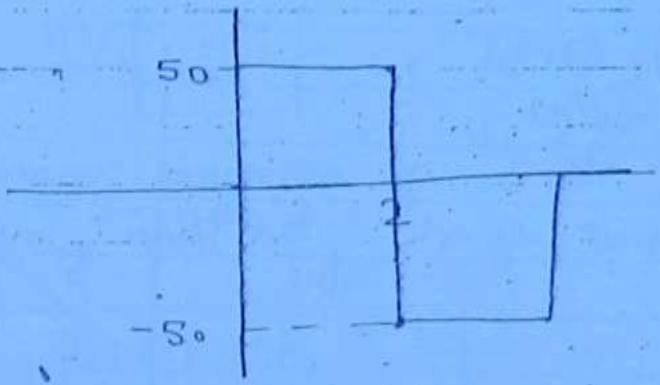
$\frac{dh}{dt} =$



$\therefore = 10[\delta(t) + \delta(t-2)]$

$f(t) \circledast \frac{dh}{dt} = 10 f(t) - 10 f(t-2)$

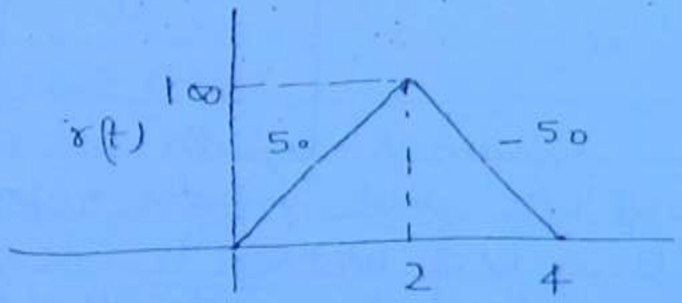
$\frac{dr}{dt}$



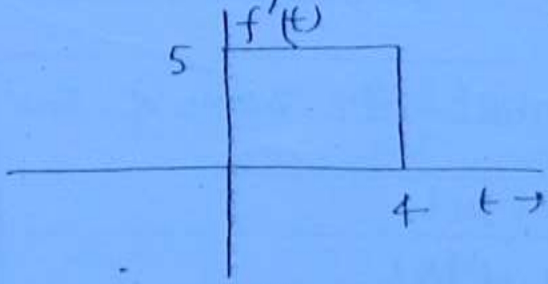
$50t + (-100$

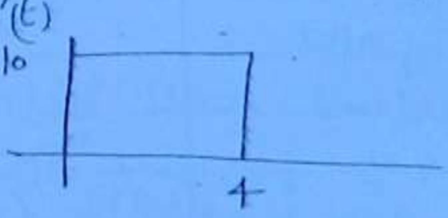
$-50t$

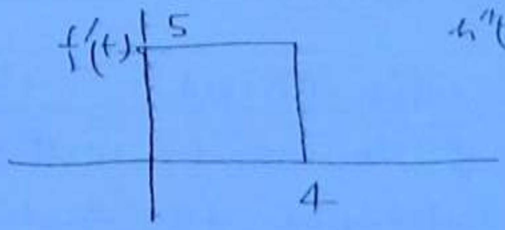
$\int_{-\infty}^{t} \frac{dr}{dt} dt =$

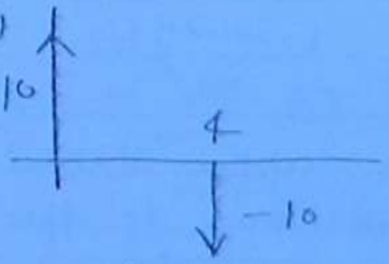


Q. Calculate the convolution of following signal

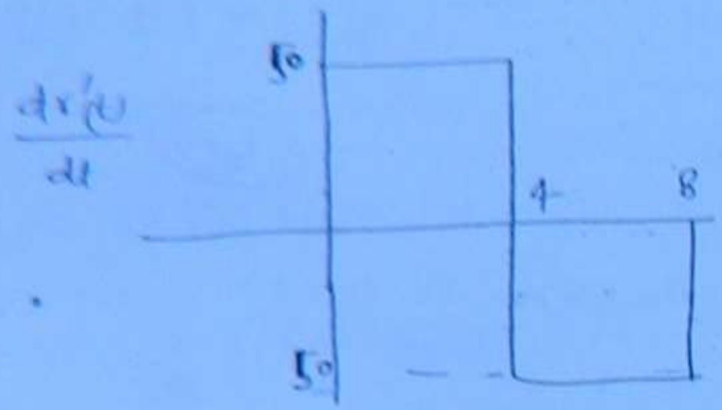




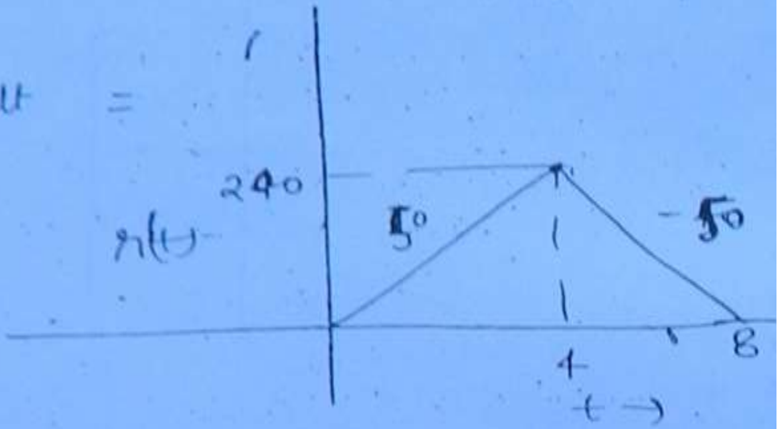




$f'(t) \circledast h''(t) = 10 f'(t) - 10 f'(t-4)$



$$\therefore \; r'(t) = \int_{-\infty}^{t} \frac{dr'(t)}{dt}\, dt =$$



* Width of resultant signal will be always some of widths of i/p and impulse respons h(t)

i.e. $\underset{\text{width of } r(t)}{w} = \underset{\text{width of } f(t)}{w_1} + \underset{\text{width of } h(t)}{w_2}$

⊛ Lower extend will be equal to some of lower extends of f(t) and h(t)

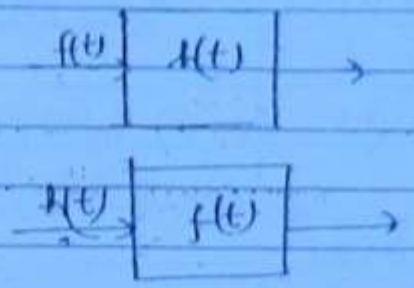
$\underset{\text{lower end of } r(t)}{l} = \underset{\text{lower end of } f(t)}{l_1} + \underset{\text{lower end of } h(t)}{l_2}$

Similarly upper extend of resultant will be equal to some of upper extends of f(t) & h(t)

$$\boxed{u = u_1 + u_2}$$

Commutative Property :→

$$f(t) \otimes h(t) = h(t) \otimes f(t)$$



$f(t) \otimes h(t) = r(t)$

$\overset{\shortparallel}{\text{same}}$

$h(t) \otimes f(t) = r(t)$

⊗ Interchanging positions of i/p and impulse response for an LTI system does not change response of the system

$$\boxed{\begin{aligned} \frac{dr}{dt} &= f(t) \otimes \frac{dh}{dt} \\ \frac{dr}{dt} &= h(t) \otimes \frac{d f(t)}{dt} \end{aligned}}$$

$$\boxed{\begin{aligned} \frac{d^2 r}{dt^2} &= h(t) \otimes \frac{d^2 f(t)}{dt} \\ &= f(t) \otimes \frac{d^2 h(t)}{dt^2} \\ &= \frac{df(t)}{dt} \otimes \frac{d h(t)}{dt} \\ &= \frac{d h(t)}{dt} \otimes \frac{d f(t)}{dt} \end{aligned}}$$

$$\frac{d^m r}{dt^m} = h(t) \otimes \frac{d^m f(t)}{dt^m} = \frac{d^m h(t)}{dt^m} \otimes f(t)$$

$$= \frac{d^{m-1}}{dt^{m-1}} h(t) \otimes \frac{d f(t)}{dt}$$

$$= \frac{d^n f(t)}{dt^n} \otimes \frac{d^p h(t)}{dt^p}$$

$$= \frac{d^{m-2}}{dt^{m-2}} h(t) \otimes \frac{d^2 f}{dt^2}$$

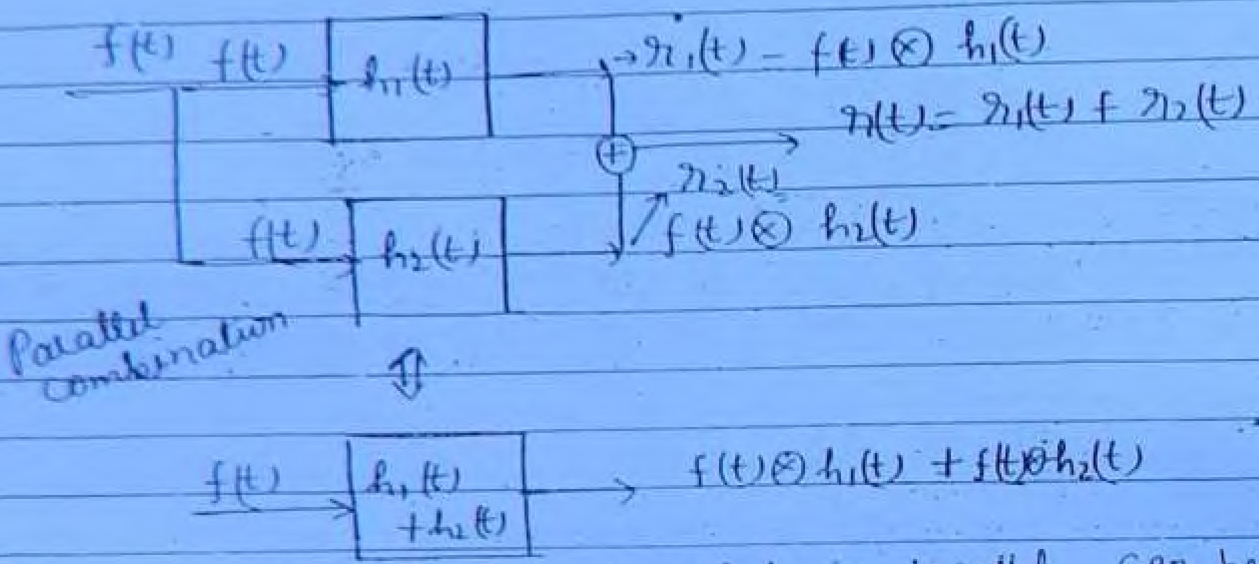
where $\boxed{(n+p) = m}$

$$= d^{m-1} f(t) \otimes \frac{dh}{} = \cdots$$

$a\times[b+c] = a\times b + a\times c$

distributive property →

$$h(t)\otimes[f_1(t)+f_2(t)] = h(t)\otimes f_1(t) + h(t)\otimes f_2(t)$$

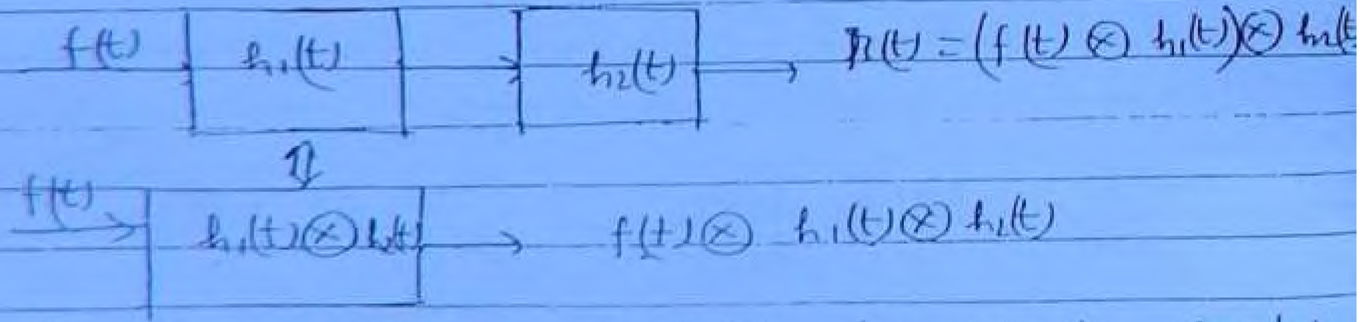


⊗ Any no of systems connected in parallel can be combined into a single system whose impulse response is some of all the individual impulse responses.

Associative Property :→

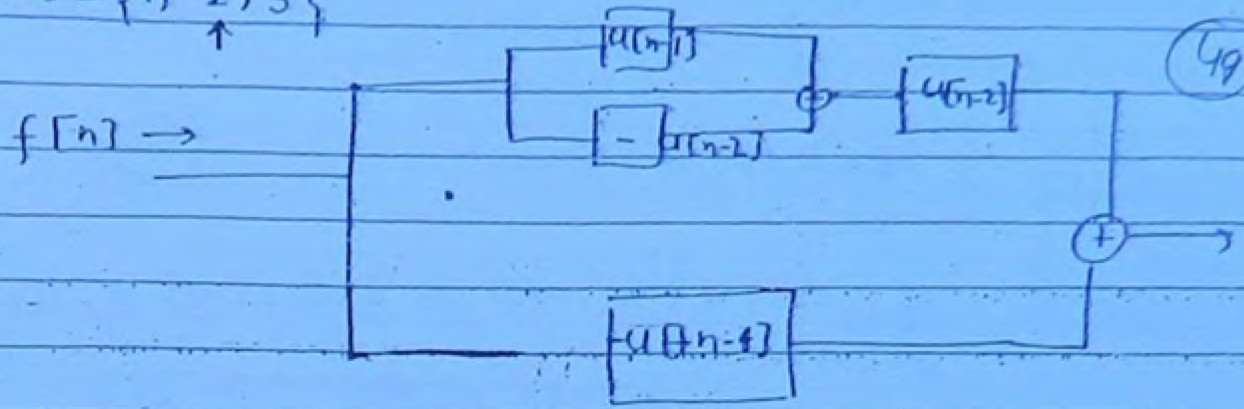
$f(t)$, $h_1(t)$, $h_2(t)$

$$f(t)\otimes h_1(t)\otimes h_2(t) = [f(t)\otimes h_1(t)]\otimes h_2(t)$$



⊗ Any no of systems connected in cascade can be combined into a single system whose impulse response is convolution of all the individual impulse responses.

Q. What is the response of following system to an i/p $f[n] = \{1, \underset{\uparrow}{2}, 3\}$



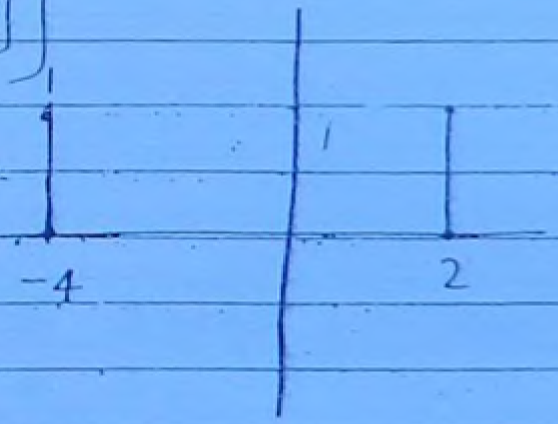
$$x[n] = f[n] \otimes \left[-u[+n-4] + \{ u[n-1] - u[n-2] \} \otimes u[n-2] \right]$$

$$= f[n] \otimes \left[-u[+n-4] + \delta[n-1] \otimes u[n-2] \right]$$

$$= f[n] \otimes \left[-u[-n-4] + u[n-3] \right]$$

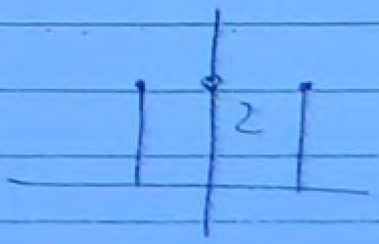
$$= f[n] \otimes \left[\delta[n-3] \right]$$

$$= f[n-3]$$



$\{ x[n]$

$f[n] \otimes u[n-2] \}$



$$\xrightarrow{f[n]} \boxed{h[n]} \longrightarrow \cancel{m} \sum_{k=-\infty}^{\infty} f[k]\, h[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, f[n-k]$$

$$r[n] = \sum_{k=-\infty}^{\infty} h[k]\, f[n-k]$$

$$= \sum_{k=-\infty}^{-1} h[k]\, f[n-k] + h[0]\, f[n] + \sum_{k=1}^{\infty} h[k]\, f[n-k]$$

for system to be causal $\boxed{h[n] = 0 \quad n<0}$

$$h(t) = 0 \quad t<0$$

$$\xrightarrow{f(t)} \boxed{h(t)} \longrightarrow \int_{-\infty}^{\infty} h(\tau)\, f(t-\tau)\, d\tau \qquad t-\tau = m$$

$\downarrow$

causal system

$$\int_{0}^{\infty} h(\tau)\, f(t-\tau)\, d\tau \qquad \tau = (t-m)$$

$$= \int_{t}^{-\infty} h(t-m)\, f(m)\, -dm$$

$$\cancel{\int_{0}^{\infty} f(\tau)\, h\, t}$$

$$r(t) = \int_{-\infty}^{t} h(t-\tau)\, f(\tau)\, d\tau$$

both are causal f(t) & system is causal

$\downarrow$

causal signal

$$\boxed{\begin{aligned} r(t) &= \int_{0}^{t} \cancel{b}\, f(\tau)\, h(t-\tau)\, d\tau \qquad \cancel{t-\tau} = m \\ r(t) &= \int_{0}^{t} \cancel{b}\, h(\tau)\, f(t-\tau)\, d\tau \end{aligned}}$$

$f(t) \longrightarrow 0 \quad \underline{t < 2}$

$$r(t) = \int^{(t-2)} f(\tau)\, h(t-\tau)\, d\tau$$

$$= (t+2) \quad f(t+2-\tau)$$

$(t+2-\tau) = t-M$

$2-\tau = -M$

(-2)

(t)

$= \int^{(t+2)} ~~f[2+M]\, h(t-M)~~\, dM$

(*). h(t) is not equal to a impulse function it is representing a dynamic system.

$$f(t) \rightarrow \boxed{} \rightarrow r(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau$$

$|f(t)| \leq M$

$$= \int_{-\infty}^{\infty} f(t-\tau)\, h(\tau)\, d\tau$$

$$= \left| \int_{-\infty}^{\infty} f(t-\tau)\, h(\tau)\, d\tau \right| < \infty$$

or

$$\left| \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau \right| < \infty$$

$$\int_{-\infty}^{\infty} |h(\tau)|\, |f(t-\tau)|\, d\tau \leq \infty$$

$$\int_{-\infty}^{\infty} |h(\tau)|\, M\, d\tau < \infty$$

$$\boxed{\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty} \rightarrow \text{system to be stable}$$

$$\sum_{K=-\infty}^{\infty} |h(K)| < \infty$$

$$\boxed{\sum_{n=-\infty}^{\infty} |h[n]| < \infty}$$

$\}\, h(t) = \sin t$ → unstable system   oscillatory

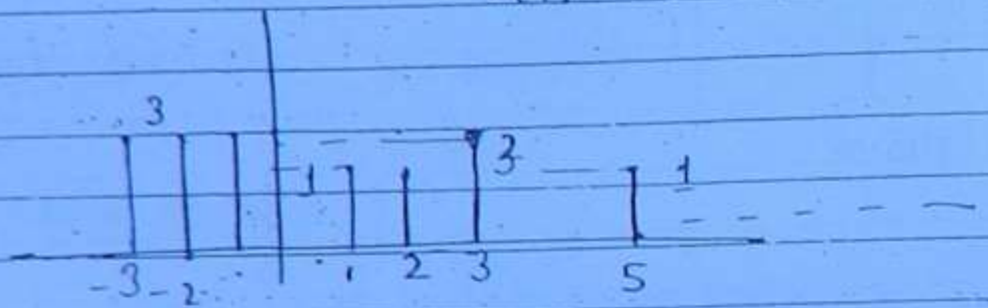
$h(t) = e^{-at}u(t)$ → $1/a$ area   stable system.

$h[n] = u[n]$ → unstable

$\}\, h[n] = (-1)^n u[n]$ → unstable · oscillatory

Q. A discrete time system is defined by the impulse response $h[n] = 3u[n+3] - 2u[n-1] + 2u[n-3] - 2u[n-5]$

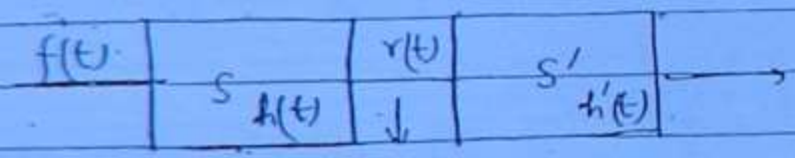
$h[-1] = 3$ → not causal



unstable, not causal

21st oct 10

Invertible or non invertible →



$[f(t) \otimes h(t)] \otimes h'(t) = f(t)$

$= f(t) \otimes [h(t) \otimes h'(t)]$

$= f(t) \otimes \delta(t)$

$= f(t)$

condition for invertibility

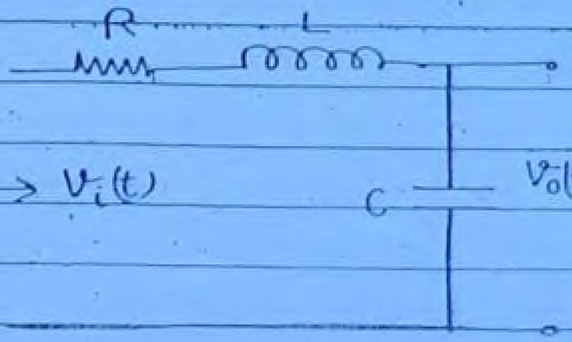
$\boxed{h(t) \otimes h'(t) = \delta(t)}$

Q. A discrete time system's impulse response is defined as $h[n] = \alpha^n u[n] + \beta^n u[-n-1]$ the system is stable only if.

$0 < |\alpha| < 1$ ✓ ⑤③

$|\beta| > 1$ ✓

⊗



$$LC\frac{d^2 v_0}{dt^2} + RC\frac{dv_0}{dt} + v_0(t) = v_i(t)$$

⊛ A system is general can be represented by a differential eq^n given below

$$a_2\frac{d^2y}{dt^2} + a_1\frac{dy}{dt} + a_0 y = b_1\frac{dx}{dt} + b_0 x(t)$$

In above differential eq^n there are no product terms of the i/p $x(t)$ or of o/p $y(t)$ are their darivatives i.e. there are no terms like $x^2(t)$, $x^3(t)$, $\left(\frac{dy}{dt}\right)^2$ $\left(\frac{d^2}{d}\right.$

$\left(\frac{dx}{dt}\right)^2$, $\left(\frac{d^2x}{dt^2}\right)^2$, $[y(t)]^2$, $\left(\frac{dx}{dt}\frac{d^2x}{dt^2}\right)$ ---

such a differential equation is called a linear differential and it directly corresponds to a linear system In ~~the~~

⊛ In the above differential eq^n if all the coefficients a_2, a_1, a_0, b_1, b_0 are ~~const~~ independent of time (i.e. constants) then the above differential eq^n can be called as a constant coefficient differential eq^n and it directly corresponds to time invariant system

Q. A system represented by following differential eqⁿ

$$2\cdot\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4t\,y(t) = 2x(t)$$

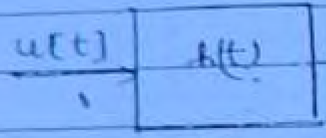
linear but time varient

$$2\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y^2(t) = 2x(t)$$

Non linear, but time invarient

$$2\frac{d^2y}{dt^2} + \sin t\cdot\frac{dy}{dt} + y(t) = 2x(t)$$

Linear, time varient



$$x(t) = \int_{-\infty}^{\infty} u(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\,u(t-\tau)\,d\tau$$

$$x(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

∴ system is linear

∴ response to u(t) ⊙ $x(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$

$$h(t) = \frac{d\,x(t)}{dt} = \frac{dS(t)}{dt}$$

S(t) is step response

Q. The following four signal define step responses & impulse responses of continuous time LTI systems

(i) $(5-4e^{-2t})\,u(t)$

(ii) $(4-3e^{-2t})\,u(t)$

(iii) $\cancel{5}\,\delta(t) + 8e^{2t}u(t)$

(iv) $2\delta(t) + 6e^{-2t}u(t)$

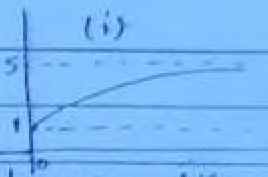
of this which two signals corresponds to step response & impulse response of the same LTI system ?

Ans

(i) & (iii)

(i) step response

(iii) is impulse response



$h(t) = 5\delta(t) + (8e^{-2t}u(t))$

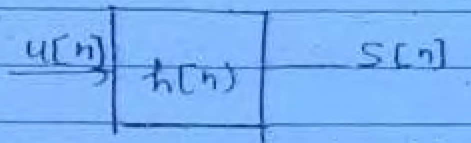
$s(t) = \int_{-\infty}^{t} h(\tau)d\tau = 5u(t) + 8\int_{0}^{t} 8e^{-2\tau}u(\tau)d\tau$

$= 5u(t) + 8\int_{0}^{t} e^{-2\tau}d\tau$

$= 5u(t) - 4(e^{-2t} - 1)$

$$S(t) = (5 - 4e^{-2t})\,u(t)$$

$$h(t) = \frac{ds(t)}{dt} = 5\delta(t) - 4\delta(t) + 8e^{-2t}u(t)$$
$$= \delta(t) + 8e^{-2t}u(t)$$



$h[n] = S[n] - S[n-1]$

impulse response → $S[n]$ step response

$$S[n] = \sum_{K=-\infty}^{n} 6\,h[n]$$

Q. For a discrete time system, the response to a step i/p is known to $S[n]\begin{Bmatrix} 1, & \underset{\uparrow}{2}, & 1 \end{Bmatrix}$ find ~~h[n]~~ response of the system for $f[n] = \{\underset{\uparrow}{1}, 2;$

$$S[n-1] = \{\underset{\uparrow}{1}, 2, 1\}$$

$h[n] = S[n] - S[n-1] = \{1\ \underset{\uparrow}{1}\ -1\ -1\}$

↑ impulse response

$$f[n] = \{\underset{\uparrow}{1}, 2, 3\}$$

	1	1	-1	-1
1	1	1	-1	-1
2	2	2	-2	-2
3	3	3	-3	-3

$y[n] = \{1\ \underset{\uparrow}{3}\ 4\ 0\ -5\ -$

Ans

Eigen Function :

$$f(t) \rightarrow \boxed{A\delta(t)} \rightarrow A\,f(t)$$

* If the response of a given LTI system is same as the i/p function f(t) except for a scalar multiple, the function is defined as eigen function of the system. So, for a system with unit impulse response Aδ(t), any general signal f(t) forms an eigen function

$$f(t) \rightarrow \boxed{A\delta(t-t_0)} \rightarrow A\,f(t-t_0)$$

$$f(t+t_0) \qquad\qquad A\,f(t)$$

$$f(t) \otimes A\,\delta(t-t_0)$$

$$A\,f(t-t_0)$$

$$f(t-t_0) = f(t)$$

$$\downarrow$$

✓ f(t) should be periodic with period t_0.

⊗ For a system with impulse response $A\delta(t-t_0)$, any signal which is periodic with a period t_0 forms an eigen function.

$$\cos(\omega_0 t) \rightarrow \boxed{h(t)} \rightarrow h(t) \otimes \cos\omega_0 t$$

$$= \int_{-\infty}^{\infty} h(\tau)\,\cos[\omega_0(t-\tau)]\,d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\left[\left(\cos\omega_0 t\,\cos\omega_0\tau\right) + \right.$$

$$d\tau$$

$$\int_{-\infty}^{\infty} h(\tau)\,\cancel{\cos}\,\sin\omega_0 t\,\sin\omega_0\tau\,d\tau$$

$$y(t) = \cos\omega_0 t \int_{-\infty}^{\infty} h(\tau)\cos\omega_0\tau\,d\tau$$

$$+ \sin\omega_0 t \int_{-\infty}^{\infty} h(\tau)\,\sin\omega_0\tau\,d\tau$$

even signal

scaler (constant)

$$\int_{-\infty}^{\infty} h(\tau)\cos\omega_0\tau\, d\tau = A$$

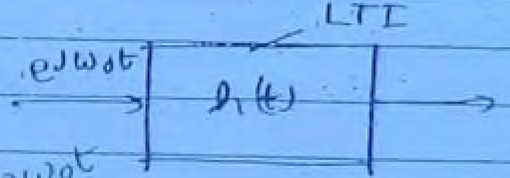
$$\int_{-\infty}^{\infty} h(\tau)\sin\omega_0\tau\, d\tau = 0$$

should be even signal

condition for $\cos\omega_0 t$ to becoms eigen function of system.

(57)

⊗ For a system with general impulse response h(t), if $\cos\omega_0 t$ is to formed eigen function then impulse response h(t) must be an even function of time.



$\rightarrow e^{j\omega_0 t} \otimes h(t)$

$$= \int_{\infty}^{\infty} h(\tau)\, e^{j\omega_0(t-\tau)}\, d\tau =$$

$$= e^{j\omega_0 t}\int_{-\infty}^{\infty} h(\tau)\, e^{-j\omega_0\tau}\, d\tau$$

$$= e^{j\omega_0 t}\int_{-\infty}^{\infty} h(\tau)\left[\cos\omega_0\tau - j\sin\omega_0\tau\right]$$

$$= e^{j\omega_0 t}\left[\int_{-\infty}^{\infty} h(\tau)\cos\omega_0\tau\, d\tau - j\int_{-\infty}^{\infty} h(\tau)\sin\omega_0\tau\, d\tau\right.$$

$e^{j\omega_0 t}$ always forms an eigen function for a system having impulse response h(t)

$h(\tau) \rightarrow$ should be even

$h(t) \rightarrow$ should be an even function of time to then $e^{j\omega_0 t}$ can form eigen function for system having impulse response h(t) if h(t) is an even function of time or ~~h(t) should be even conjugate function~~

$$e^{j\omega_0 t} \longrightarrow H(j\omega_0)\, e^{j\omega_0 t}$$

$$C_1 e^{j\omega_0 t} \longrightarrow C_1 H(j\omega_0)\, e^{j\omega_0 t}$$

$$C_2 e^{j\omega_1 t} \longrightarrow C_2 H(j\omega_1)\, e^{j\omega_1 t}$$

If $f(t) = \sum C_n e^{jn\omega_0 t} \Rightarrow r(t) \rightarrow \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} H(jn\omega_0)$

④ Signal $e^{j\omega_0 t}$ forms an eigen function for any general LTI system with impulse response, response of any LTI system when i/p is $e^{j\omega_0 t}$ can be written as $e^{j\omega_0 t} \cdot H(j\omega_0)$ where $H(j\omega_0)$ is defined as

$$H(j\omega_0) = \int_{-\infty}^{\infty} h(\tau) \cdot e^{-j\omega_0 \tau} d\tau$$

If we can now express any signal $f(t)$ as some of complex exponentials, response also will be some of no of complex exponential signal multiplyed by a suitable value

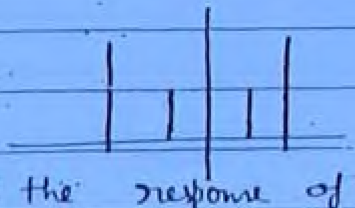
i.e. $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$ will have a response

$$r(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} H(jn\omega_0)$$

where $H(jn\omega_0) = \int_{-\infty}^{\infty} h(\tau) e^{-jn\omega_0 \tau} d\tau$

Q. A discrete time system with an impulse response

$h[n] = 2 \quad n = 2, -2$
$\quad\quad\quad 1 \quad n = 1, -1$
$\quad\quad\quad 0 \quad$ otherwis



given an i/p $f[n] = e^{jn\pi/2}$ find the response of the system

$$r[n] = \sum_{K=-\infty}^{\infty} h[K] f[n-K] = \sum_{K=-\infty}^{\infty} h[K] e^{j(n-K)\pi/2}$$

$$= e^{jn\pi/2} \sum_{K=-\infty}^{\infty} h[K] e^{-jK\pi/2}$$

$$= e^{jn\pi/2} \left[2 \cdot e^{j\pi} + 1 e^{j\pi/2} + 1 e^{-j\pi/2} + 2 e^{-j\pi} \right]$$

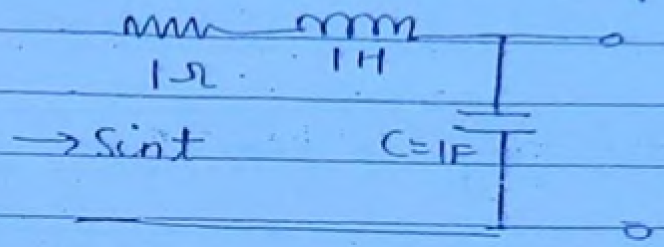
$$= e^{jn\pi/2} [-4]$$

$r[n] = -4 \, e^{jn\pi/2}$

④ eigen value $e^{jn\pi/2}$ and eigen function

$\frac{2\pi}{\pi/2} = ④$

Q. Find the response of following ckt to an i/P sint



59

$$\cancel{i(t)} = \frac{\cancel{\sin t}}{\cancel{1}} \qquad \sin t = \frac{e^{jt} - e^{-jt}}{2j}$$

$$= \frac{\sin t}{1 + j + \frac{1}{j\times 1}}$$

$$i(t) = \sin t$$

$$\therefore V(t) = \frac{1}{j}\sin t = \sin(t - \pi/2) = -\cos t$$

$$= -j\sin t \qquad -\sin(t + 3\pi/2)$$

$$\boxed{v(t) = \sin t \angle 3\pi/2}$$

o/P i/P

$$L\cdot C\cdot\frac{d^2 v_1(t)}{dt^2} + RC\frac{dv_1(t)}{dt} + v_1(t) = v_i(t)$$

$$1\cdot\frac{d^2}{dt^2}\sin t + 1\cdot\frac{d}{dt}\sin t + \sin t = v_i(t)$$

LTI system → $-\cancel{\sin t} + \cos t + \cancel{\sin t} = V_0(t)$

$$V_0(t) = \cos t$$

$$V_0(t) = \cancel{\cos} - \sin(\pi/2 + t)$$

$$= \sin(t - \pi/2)$$

$$H(j\omega_0) = -j$$

$$H(j\cdot 1) = -j$$

$$H(j\omega_0) = \frac{1}{(j\omega_0)^2 + j\omega_0 + 1}$$

inner product

$\bar{x} \cdot \bar{y} = 0$

$[\bar{x}, \bar{y}] \rightarrow$ orthogonal

independent

then we can express any a vector inform of $\bar{x}$ & $\bar{y}$ vectors

Similary $f(t)$, $g(t)$

$$\int_0^T f(t)\, g(t)\, dt = 0$$

in interval→$(0, T)$, $f(t)$, $g(t)$ are orthogonal

$$\int_0^{2\pi/\omega_0} \sin m\omega_0 t \cdot \sin n\omega_0 t \, dt$$

m, n integer (+ve) ≥ 0 or −ve integer ≤ 0
and $m \neq n$

then above integral will be zero always.

then $\sin m\omega_0 t$ is orthogatal $\sin n\omega_0 t$ ~~is orth~~ in interval. $(0, 2\pi/\omega_0)$

$$\int_0^{2\pi/\omega_0} \cos m\omega_0 t \; \cos n\omega_0 t \, dt$$

if $m \neq n$ and $m \geq 0$ or $m \leq 0$
condition on m & n
and $n \geq 0$ or $n \leq 0$
m and n are integers
then $\cos m\omega_0 t$ & $\cos n\omega_0 t$ both will be orthogonal in interval $(0, 2\pi/\omega_0)$

~~Same will be true for~~

$$\int_0^{2\pi/\omega_0} \sin K\omega_0 t \; \cos K\omega_0 t \, dt = 0$$

if K is any integer other than 0

so we can define any f(t) in terms of independent function $\cos k\omega_0 t$ & $\sin k\omega$

$$f(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + ----$$
$$+ b_1 \sin\omega_0 t + b_2 \sin 2\omega_0 t + ---$$

(6)

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$\downarrow$ f(t) is periodic with period $= \frac{2\pi}{\omega_0}$

22nd oct 10

$$f(t) = a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2 + b_n^2} \cos(\overset{n}{\omega_0} t - \phi_n)$$

$$\boxed{f(t) = \sum_{n=0}^{\infty} \gamma_n \cos(n\omega_0 t - \phi_n)}$$

⊕ Fourier series represents the information of the signal f(t) as amplitude and angles at different angular frequencies which are integer multiples of a fundamental angular frequency ω_0 i.e. $n\omega_0$.

where

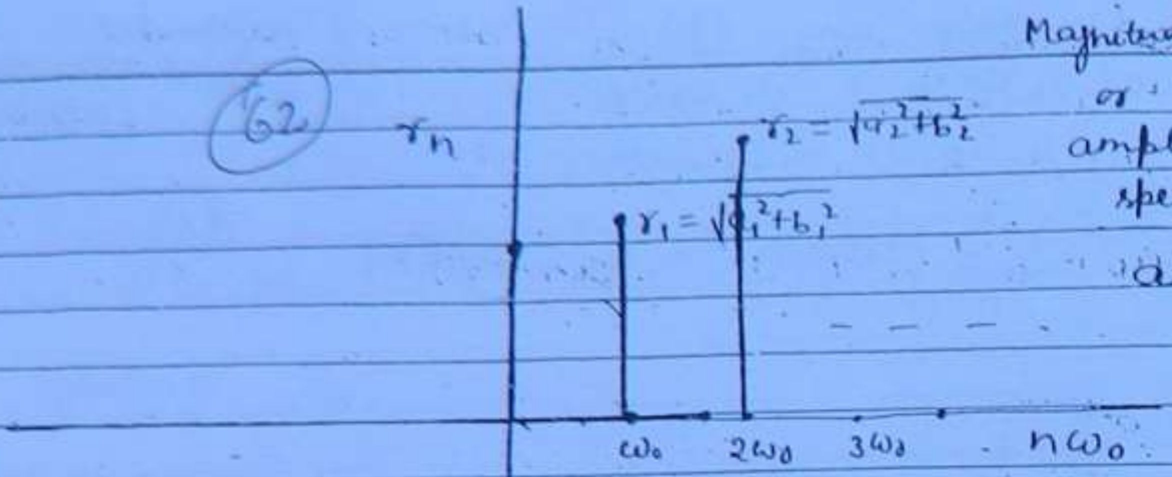
$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} 1.f(t)\, dt$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos n\omega_0 t\, dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin n\omega_0 t\, dt$$

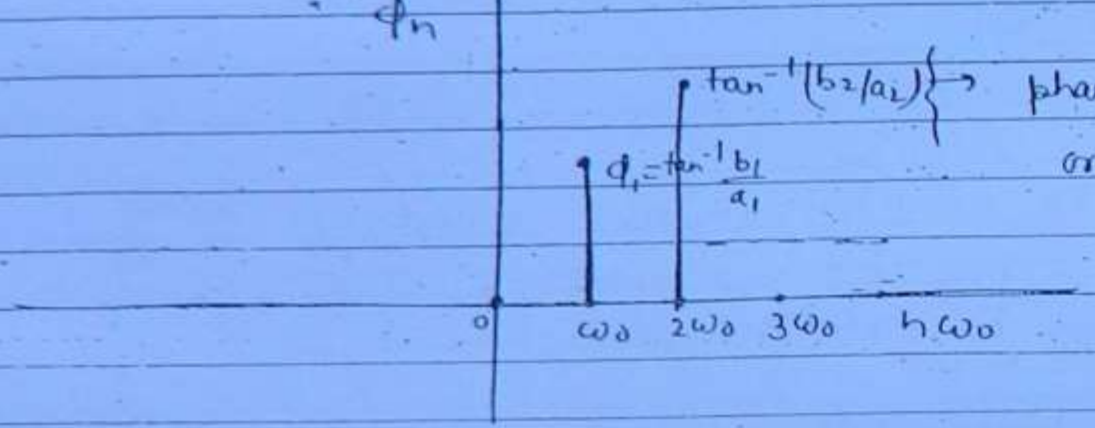
$$a[n\omega_0] = a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos n\omega_0 t\, dt$$

$$b[n\omega_0] = b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin n\omega_0 t\, dt$$



Magnitude spectrum
or
amplitude
spectrum

any graph
w.r.t. ω
frequency
is called
spectrum.



→ phase spectrum
or angle spectrum

~~Gon~~ $\cos n\omega_0 t$

$\sin n\omega_0 t$

$\{e^{jn\omega_0 t}\} \rightarrow n \rightarrow -\infty \rightarrow \infty$ integers → all have time period $\left(\frac{2\pi}{\omega_0}\right)$

$$\boxed{\int_0^T f(t)\, g^*(t)\, dt = 0}$$

→ for f(t) & g(t) → two complex valued signal to be orthogonal in interval (0, T)

$$\int_0^{2\pi/\omega_0} e^{jm\omega_0 t}\left(e^{jK\omega_0 t}\right)^* dt = 0$$

$$f(t) = C_0 + C_1 e^{j\omega_0 t} + C_2 e^{j2\omega_0 t} + C_3 e^{j3\omega_0 t} + ---$$

$$+ C_{-1} e^{-j\omega_0 t} + C_{-2} e^{-2j\omega_0 t} + ---$$

$$f(t) = C_0 + \sum_{n=1}^{\infty} \left(C_n e^{jn\omega_0 t} + C_{-n} e^{-jn\omega_0 t}\right)$$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \longrightarrow$$ Exponential fourier series

$\downarrow$

Periodic with $T = 2\pi/\omega_0 \rightarrow$ fundamental period of $f(t)$.

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega_0 t\,dt - \frac{1}{T}\int_{-T/2}^{T/2} j f(t)\sin n\omega_0 t\,dt$$

$$= \frac{1}{2}a_n - \frac{1}{2}j b_n$$

$$\boxed{C_n = \frac{1}{2}[a_n - jb_n]} \longrightarrow$$ complex in nature

$\rightarrow$ valid for only $n \neq 0$

$$\boxed{C_{-n} = \frac{1}{2}[a_n + jb_n]}.$$

$$C_n^* = \frac{1}{2}[a_n + jb_n] = C_{-n}$$

$$\boxed{C_n^* = C_{-n}}$$ providing $f(t)$ to be real.

$$C_n^* = \frac{1}{T}\int_{-T/2}^{T/2} f^*(t)\left(e^{-jn\omega_0 t}\right)^* dt$$

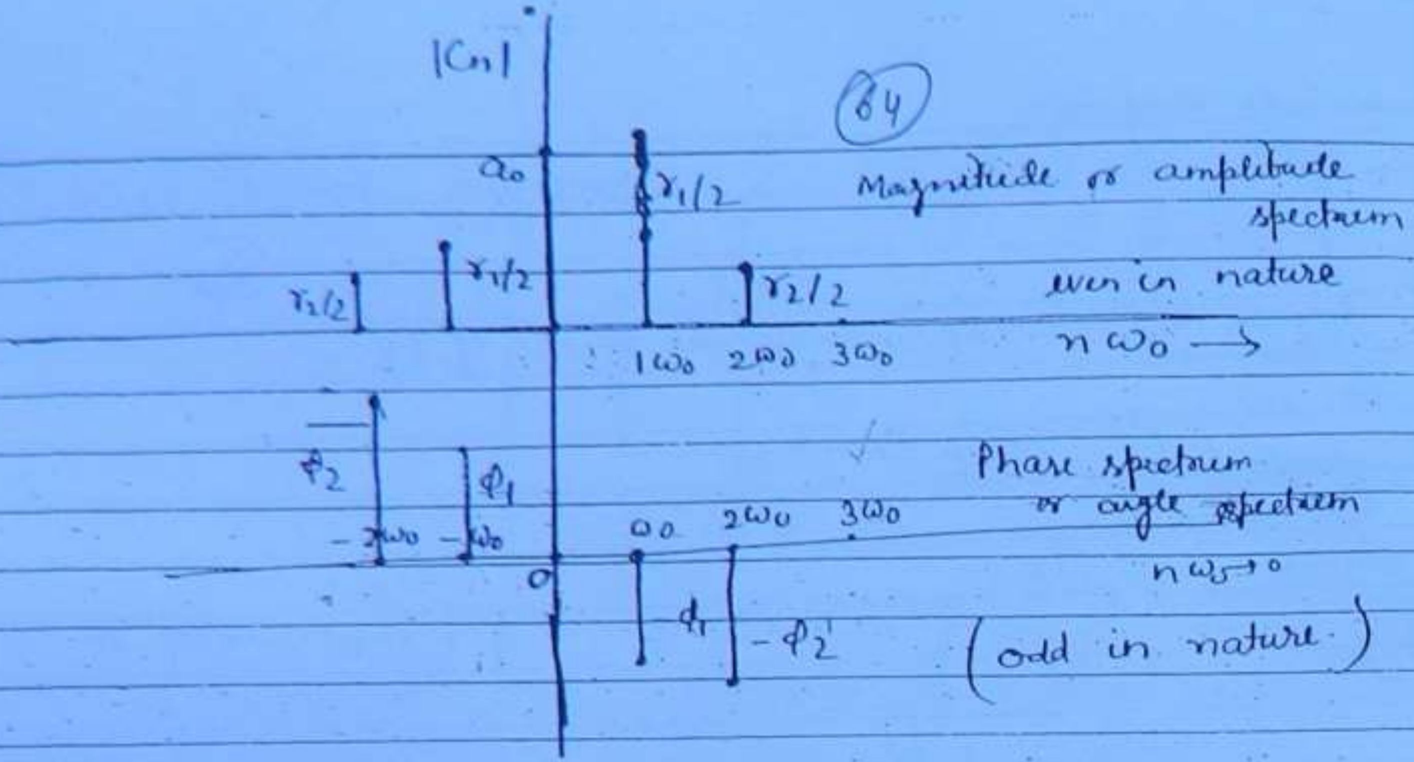
$$= \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,e^{+jn\omega_0 t}\,dt$$

$$\boxed{C_n^* = C_{-n}}$$

$$|C_n| = \frac{1}{2}\sqrt{a_n^2 + b_n^2},\quad \angle C_n = -\tan^{-1}(b_n/a_n)$$

$$= -\phi_n$$

$$|C_n| = \frac{1}{2}r_n = |C_n^*| = |C_{-n}|,\quad = -\angle C_{-n}$$



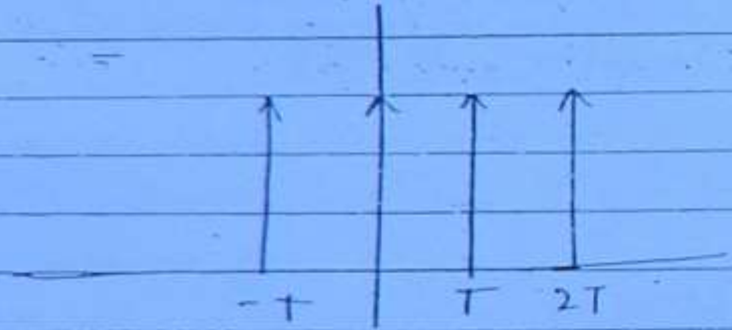
Magnitude or amplitude spectrum

even in nature

Phase spectrum or angle spectrum

(odd in nature)

Q. Find the fourier series of signal f(t) shown below



$$\delta \cdot f(t) = \sum_{K=-\infty}^{\infty} \delta(t-KT)$$

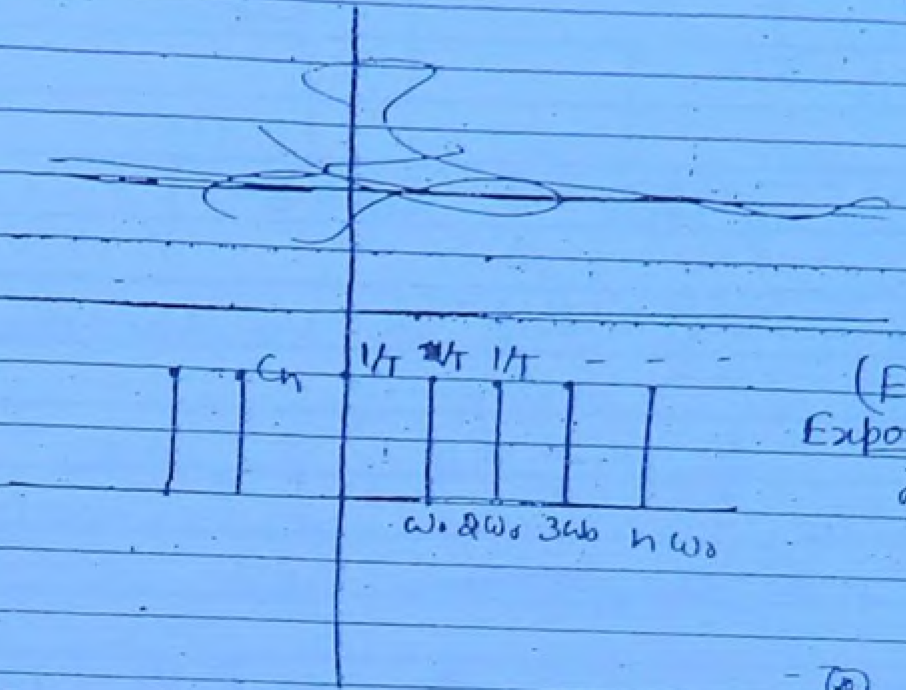
$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = \frac{1}{T}$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-jn\frac{2\pi}{T}t}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, dt = 1/T$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\omega_0 t}$$



(E.F.S.) or C.F.S.
Exponential fourier spectru
or complex fourier spect

$$C_n = \frac{a_n}{2} - j\frac{b_n}{2}$$

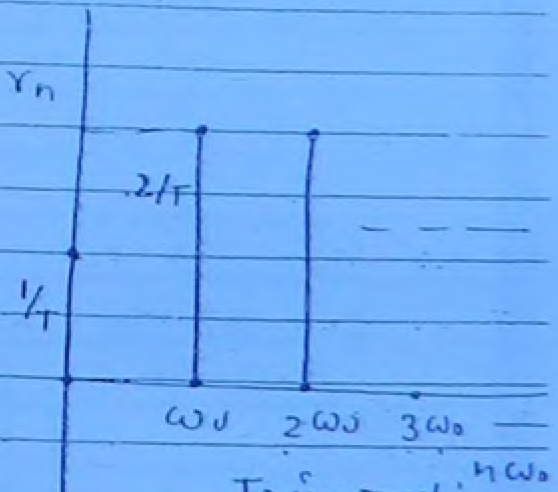
$$\boxed{C_0 = 1/T = a_0}$$

$a_n = 2/T$, $b_n = 0$, $\phi_n = 0$

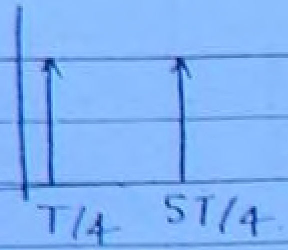
$$f(t) = \sum_{n=0}^{\infty} r_n \cos(n\omega_0 t - \phi_n)$$

$$f(t) = \frac{1}{T} + \sum_{n=}^{\infty} 2/T \cos(n\omega_0 t)$$

$$= \frac{1}{T} \sum_{n=1}^{\infty} \frac{2}{T} \cos\left(n \cdot \frac{2\pi}{T} t\right)$$



Trigonometric fourier spectru
or Real fourie
spect
(T.F.S. or R.F.S



$f(t) = \cancel{\sum\delta(t} - KT \quad \cancel{\sum\delta}(\delta -$

$$= \sum_{k=-\infty}^{\infty} \delta(t - T/4 \pm KT)$$

$$C_n = 1/T \cdot e^{-jn\frac{2\pi}{\cancel{\pi}}\cancel{\pi}/4} = \frac{1}{T} e^{-jn\pi/2} = \frac{(-1)^{n/2}}{T}$$

~~$f(t) = \frac{1}{T} + \sum_{n=1}^{\infty} \frac{2}{T} \cos n\omega_0 t$~~

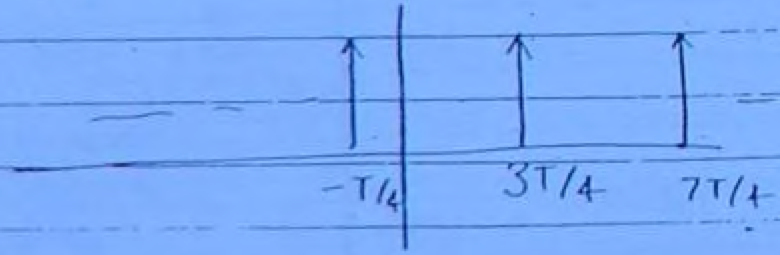
$$f(t) = \sum_{n=-\infty}^{\infty} \cancel{\frac{1}{T} C_n} e^{jn\omega_0 t} \qquad \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{-jn\pi/2} e^{jn\omega_0 t}$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} (-1)^{n/2} e^{jn\omega_0 t} \qquad \cos\frac{n\pi}{2} + j\sin\frac{n\pi}{2}$$

$$(-1)^{n/2}$$

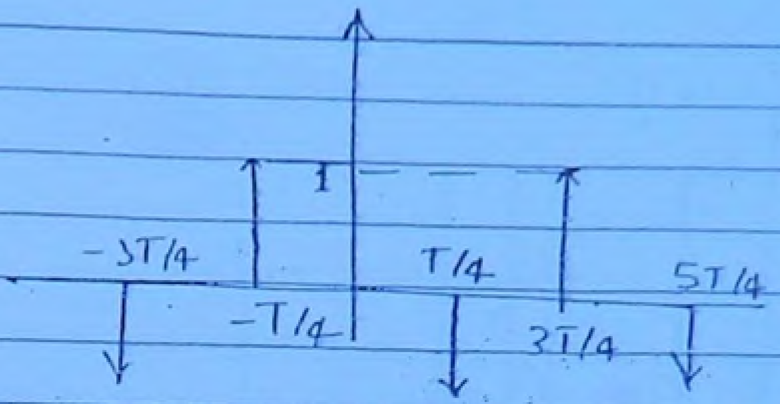
$f(t) \rightarrow C_n$

$f(t - t_0) \rightarrow C_n e^{-jn\omega_0 t_0}$



$$f(t + T/4) \rightarrow \frac{1}{T} e^{-jn\omega_0(-T/4)} \quad \text{or} \quad e^{-jn\omega_0 3T/4}$$

$$\rightarrow \frac{1}{T} e^{jn\omega_0 T/4}$$



fourier coefficient for given train of pulse →

$$C_n = \frac{1}{T} e^{jn\omega_0 T/4} - \frac{1}{T} e^{-jn\omega_0 T/4}$$

$$= \frac{1}{T} 2j \sin n\omega_0 T/4$$

$$= \frac{2j}{T} \sin \frac{n\pi}{2}$$

$f_1(t) \longrightarrow C_{1n}$

$f_2(t) \longrightarrow C_{2n}$ --

then $a f_1(t) + b f_2(t) \longrightarrow a C_{1n} + b C_{2n}$.

C_n is imaginary for odd signal and odd in nature.

Q. Find the fourier coefficients of $\sin\omega_0 t$ & $\cos\omega_0 t$

$$\sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} =$$

$C_1 = \frac{1}{2j}$ for n. 0, otherw

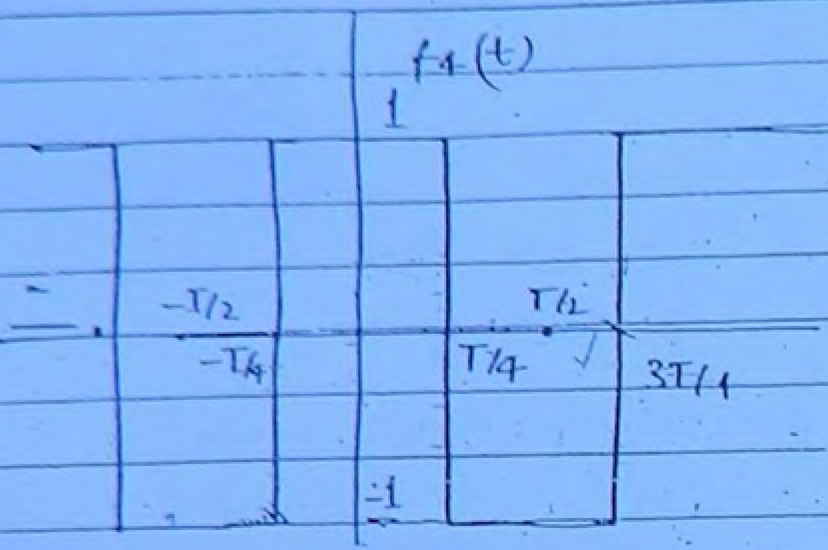
$C_{-1} = -1/2j$

$$\cos\omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$C_1 = 1/2$ { for other n 0 → else }

$C_{-1} = +1/2$

Calculate the fourier coefficient of the following signal.



$$C_n = \frac{1}{T}\int_{-T/4}^{3T/4} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_{-T/4}^{T/4} 1\, e^{-jn\omega_0 t} + \frac{1}{T}\int_{T/4}^{3T/4} e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-T/4}^{T/4} + \frac{1}{Tjn\omega_0}\left[e^{-jn\omega_0 t}\right]_{T/4}^{3T/4}$$

$$= \frac{1}{T}\left[\frac{1}{jn\omega_0}\left[e^{jn\omega_0 T/4} - e^{-jn\omega_0 T/4}\right]\right]$$

check

$$+\frac{1}{Tjn\omega_0}\left[e^{-jn\omega_0 3T/4} - e^{-jn\omega_0 T/4}\right]$$

$$= \frac{1}{Tjn\omega_0}\left[2e^{-jn\omega_0 3T/4} - 2e^{-jn\omega_0 T/4}\right]$$

$$= \frac{1\,2}{Tjn\omega_0}\left[e^{jn\omega_0 T/4} - e^{-jn\omega_0 T/4}\right]$$

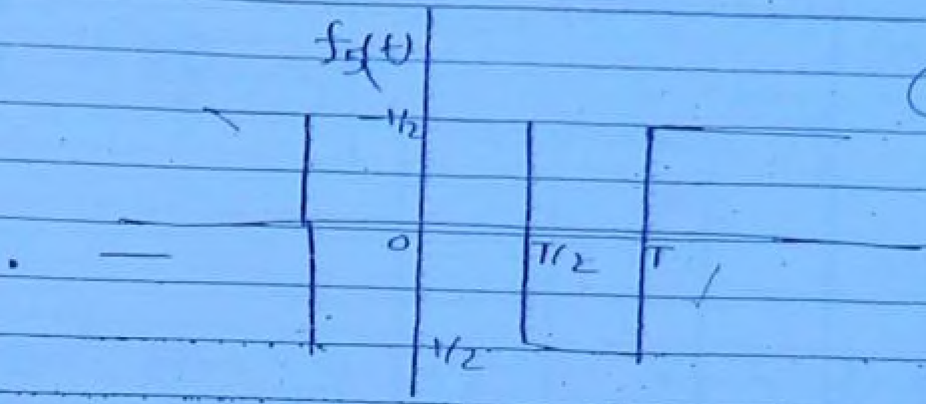
$$= \frac{1}{Tjn\omega_0}\, 2j \sin n\omega_0 T/4$$

$$C_n = \frac{4}{Tn\omega_0} \sin n\pi/2 = \frac{2}{n\pi}\sin\frac{n\pi}{2}$$

= 0 for n eve

$C_0 = 0$

(69)



f(t)

$$C_n \longrightarrow \frac{1}{n\pi} \sin\frac{n\pi}{2} \; e^{-jn\omega_0 T/4}$$

$$C_n \longrightarrow \left[\frac{1}{n\pi} \sin\frac{n\pi}{2} \cdot e^{-jn\pi/2}\right] \rightarrow \frac{1}{2jn\pi}(1-co$$

$C_0 = 0$

$\hookrightarrow$ imaginary

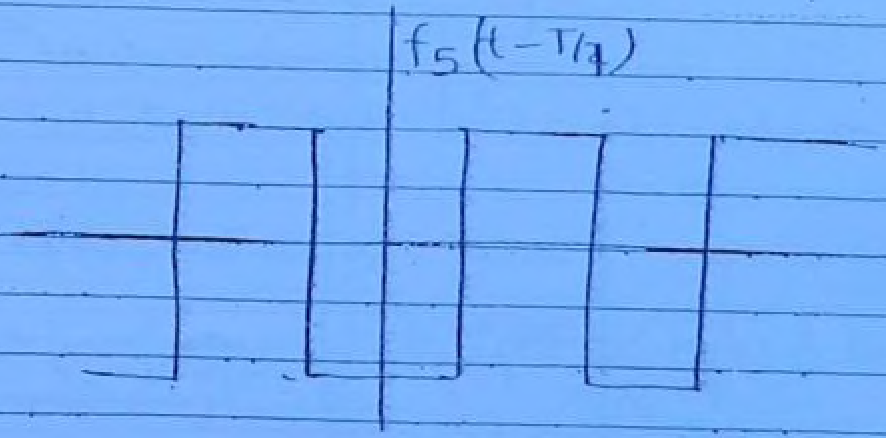
and odd for

n eve

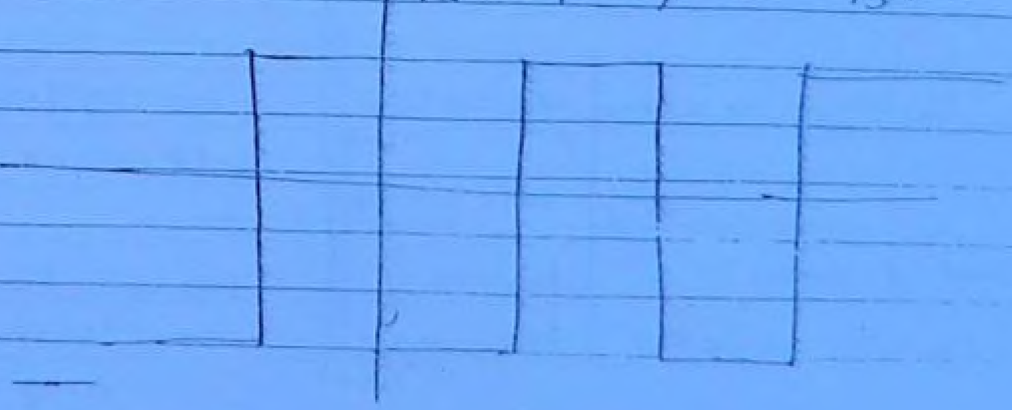
$f(t) \longrightarrow C_n$

$-\frac{d}{dt} f(t) \longrightarrow +C_n \, jn\omega_0$

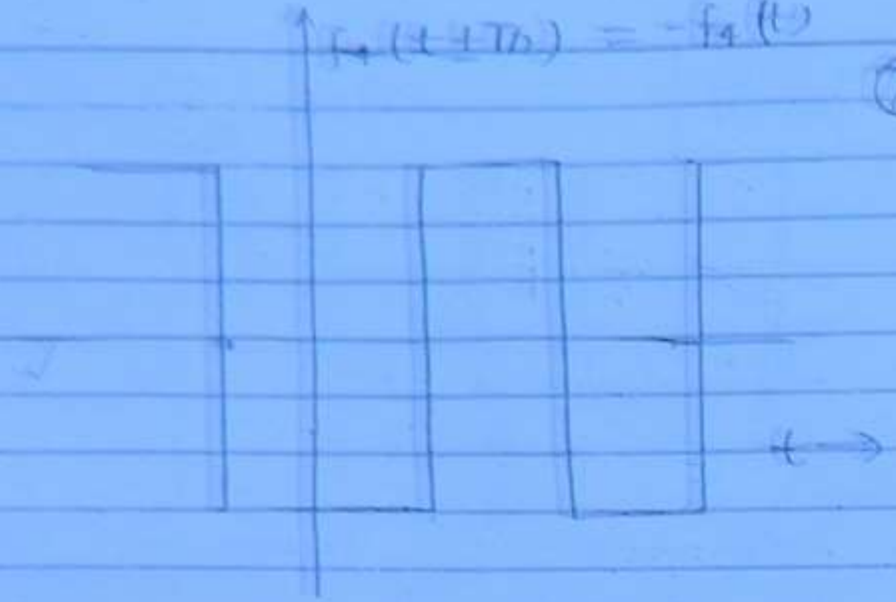
$C_n = 0$



$f_5(t+T/2) = -f_5(t)$



$f_4(t \pm T/2) = -f_4(t)$



$f(t \pm T/2) = -f(t)$ $C_n = 0$ n even

↳ hidden symmetry is called as half wave symmetry

Symmetry table

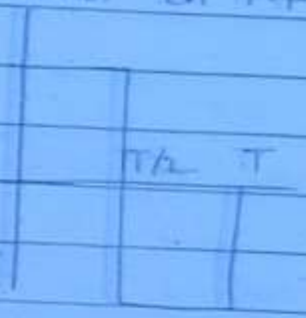
f(t)	C_n	Trigonometric Coefficient	trigo series
real and neither even nor odd	complex $C_n = C^*_{-n}$	$a_0 \neq 0, a_n \neq 0, b_n \neq 0$	$f(t) = a_0 + \sum_{n=1}^{\infty}$ [illegible] dc term, cosine term, sine terms
Even	real & even	$b_n = 0$	dc term & cosine term (may be zero or not)
odd	Purely imaginary & odd	$a_n = 0, a_0 = 0$	~~dc term~~ & Sine terms
half wave symmetry	$C_n = 0$ $n =$ even	$a_n = 0$ for $n =$ even $b_n = 0$ for $n =$ even	no dc term odd cosine & odd sine terms
even & half wave symmetrical	real & even $C_n = 0$ $n =$ even	$b_n = 0$ $a_n = 0$ n even	no dc terms odd harmonics & only cosine terms

odd & half wave symmetry	~~$a_n = 0$~~ $C_n = 0$ for n even $a_n = 0$ always	$a_n = 0$ always $b_n = 0$ for n even	no d.c term odd harmonics & only sine terms (i.e. odd sine term)

4

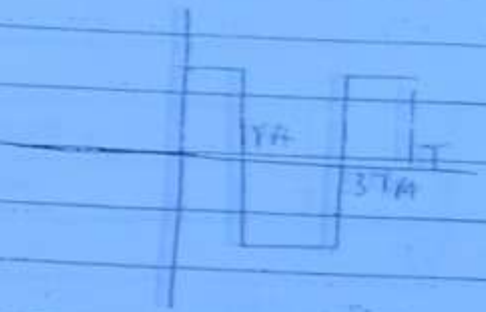
Q. ~~After~~ of following signal which of signals will have only odd sine terms in T.F.S. → defined in one time period.

(i)

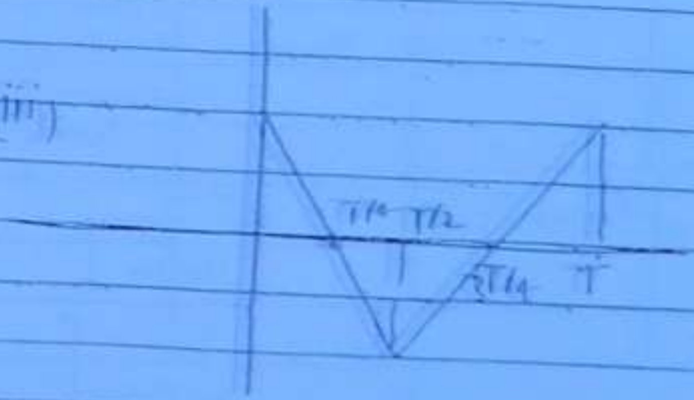


Ans (i) & (IV)

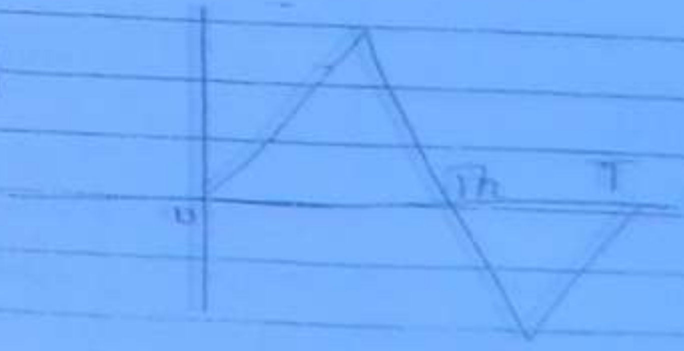
(ii)



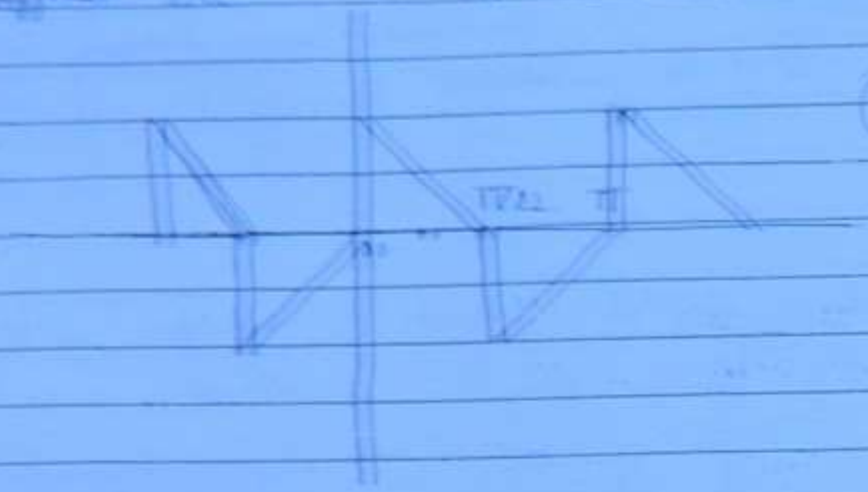
(iii)



(IV)

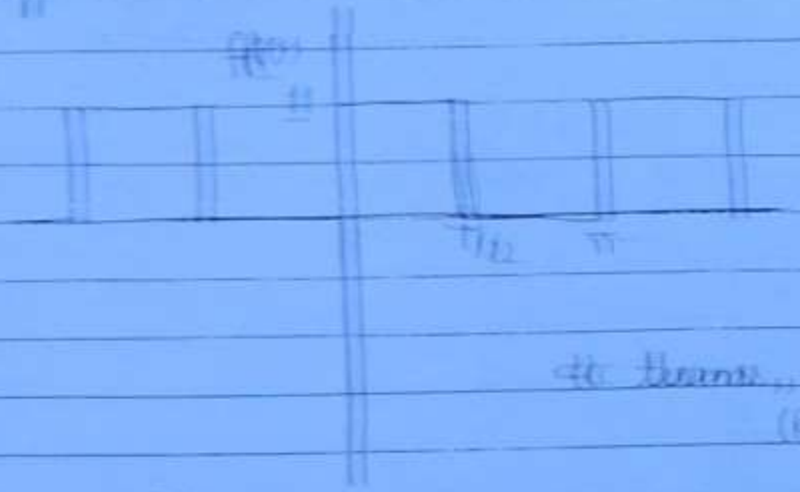


Q. A signal f(t) is defined as shown below →



→ only odd sine & cosine terms

Q. A signal f(t) is defined as below



Neither even nor odd
and not half wave
symmetry

dc terms, odd sine terms,
(odd cosine terms) Ans

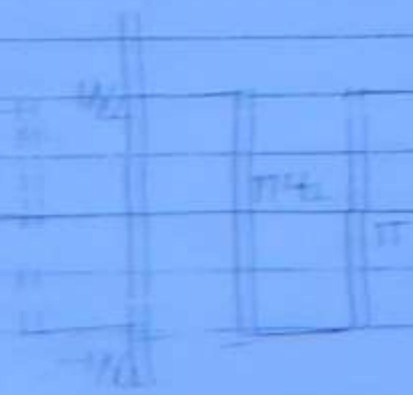
$$C_n = \frac{1}{T}\int_0^{T/2} 1 \cdot e^{-jn\omega_0 t}\,dt = \frac{1}{Tjn\omega_0}\left[1 - e^{-jn\omega_0 T/2}\right]$$

$$= \frac{1}{Tjn2\pi}\left[1 - e^{-jn\pi}\right]$$

modify f(t) − 1/2

$$C_n = \frac{1}{Tjn\omega_0}\left[1 - \cos n\pi\right]$$

$$C_n = \frac{1}{j2\pi n}\left(1 - \cos n\pi\right)$$

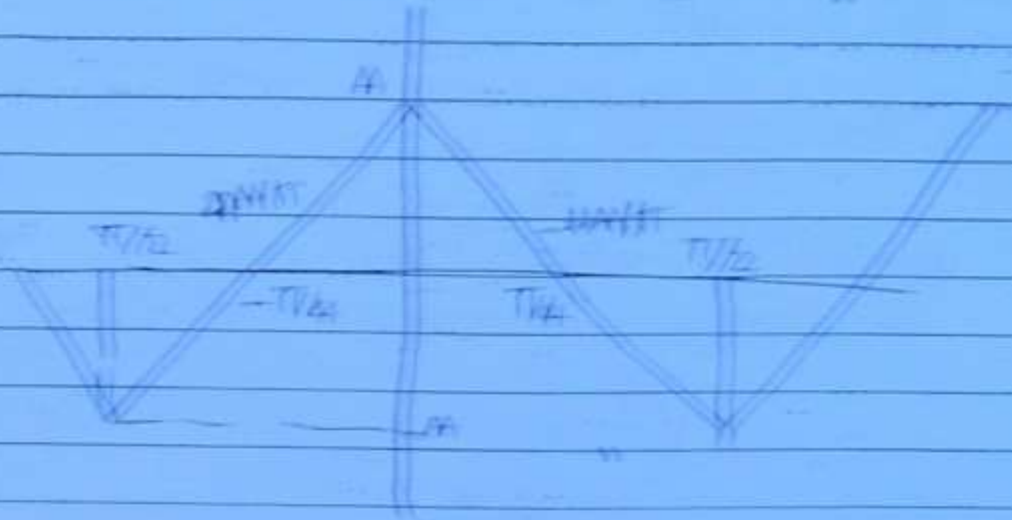


odd & half wave symmetry

Now for f(t)
dc term = 1/2
— odd sine terms (dc term 1/2 = 0)

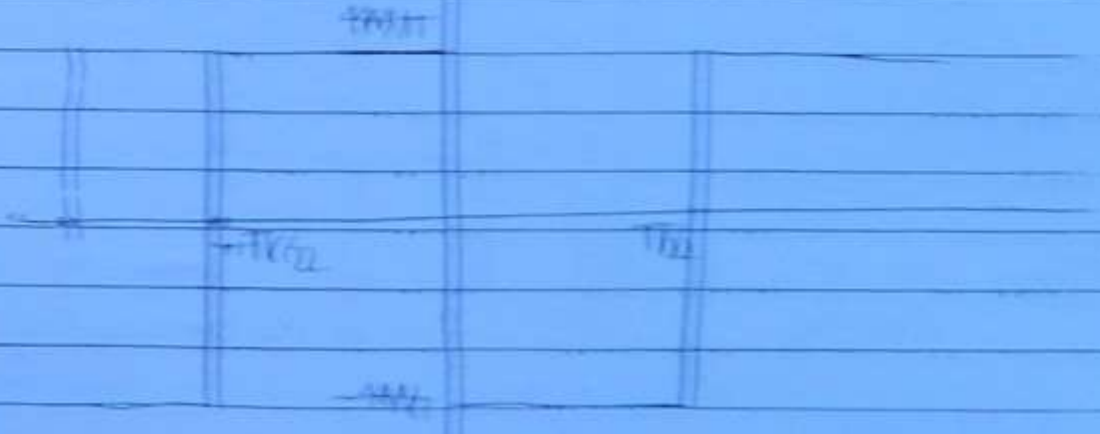
(iii) If we add some constant dc to $x(t)$ then only dc term will change and other fourier coefficient will remain same.

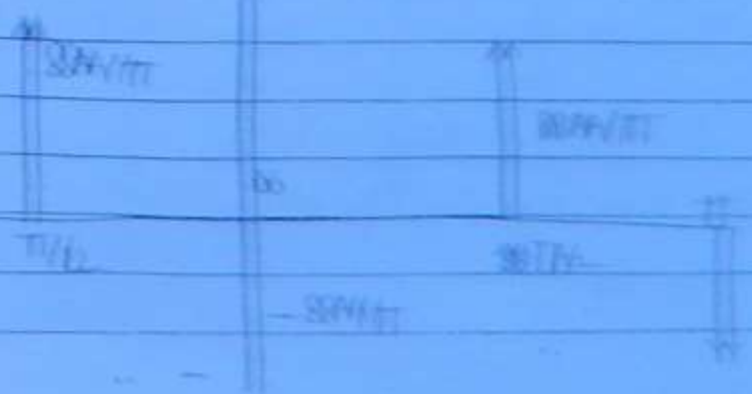
Q) Calculate the fourier coefficient of following signals.



$C_0 = 0$

$C_{an} =$





for this impulse

$$C_n = \cdot \frac{8A}{T} \cdot \frac{1}{T} e^{-jn\omega_0 T/2} - \frac{8A}{T} \cdot \frac{1}{T}$$

$$= \frac{8A}{T^2}\left[e^{-jn\omega_0 T/2} - 1\right]$$

$$= \frac{8A}{T^2}\left[e^{-jn\pi} - 1\right].$$

$$C_n = \frac{8A}{T^2}\left[\cos n\pi - 1\right]$$

fourier coefficient of f(t)

$$G \quad F_n = \frac{C_n}{(j\omega_0 n)^2}$$

$$= -\frac{8A}{T^2\,\omega_0^2\,n^2}\left[\cos n\pi - 1\right]$$

$$= \frac{8A}{T^2\omega_0^2 n^2}\left[1 - \cos n\pi\right]$$

$$\boxed{F_n = \frac{2A}{\pi^2 n^2}\left[1 - \cos n\pi\right)}$$ Ans

↓ real & even because signal is real & even.

$$F_0 = 0$$

$$F_n = \frac{2A}{\pi^2 n^2}\left[1 - \underset{\substack{\| \\ 0 \text{ for } n \text{ even}}}{(-1)^n}\right] \longrightarrow$$

and only even terms because of half wave symmetry

$$F_n \propto \frac{1}{n^2} \rightarrow F_n \propto |n|^{-2}$$

Triangular pulse

for rectangular pulse

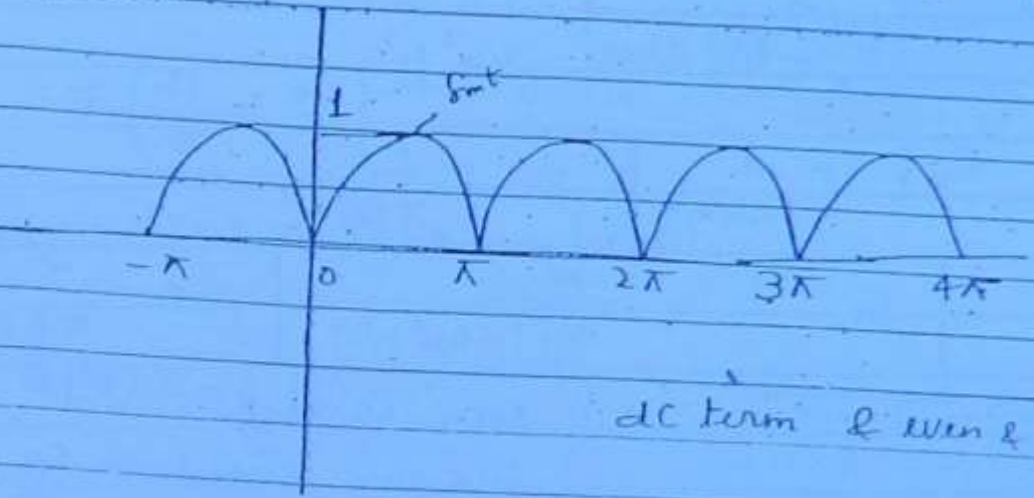
$$C_n \propto \frac{1}{n}$$

if function is varying as parabolic variation in a p
means as a function t^2.

then fourier series coefficient will be (75)

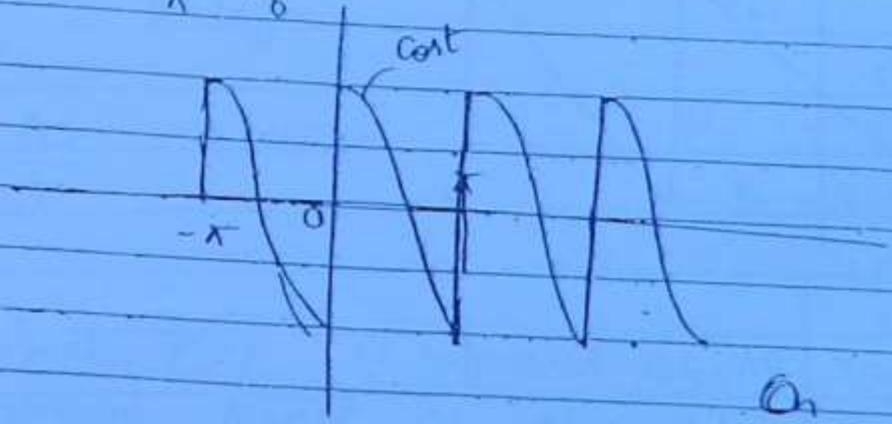
$$C_n \propto \frac{1}{n^3} \rightarrow C_n \propto |n|^{-3}$$

Q. Find the fourier coefficient of following signal

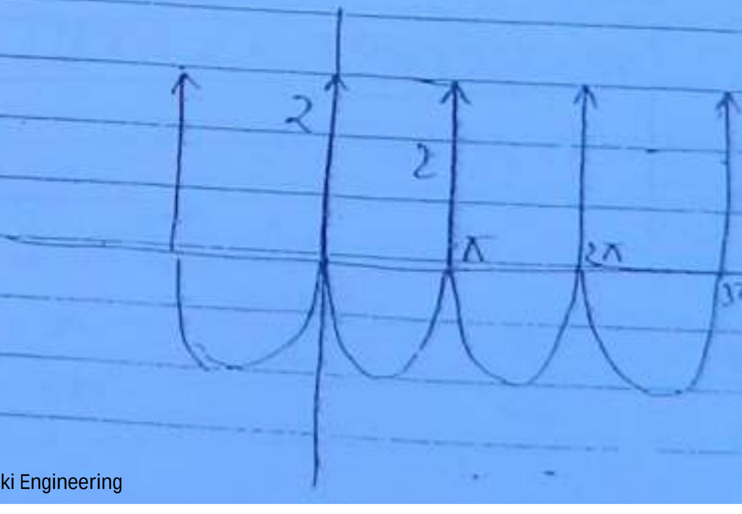


dc term & even & real

$$C_n = \frac{1}{\pi}\int_0^{\pi} \sin t \, e^{-jn\omega_0 t} \, dt$$ we can find



C_n



~~Fn = Cn for infinite~~

— Fn

~~2Fn = 2~~

π

clear
distu

$C_n = (\underset{\text{Fou}}{\text{impulse}} - F_n)$

$C_n = \left(\frac{2}{\pi} - F_n\right)$

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$F_n = \frac{C_n}{(j\omega_0 n)^2}$

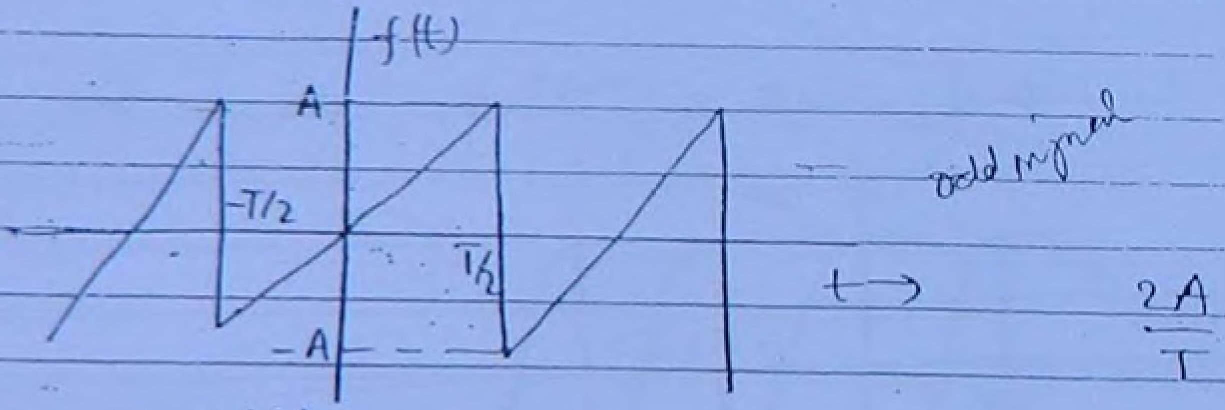
$= -\frac{2}{\pi \omega_0^2 n^2} + \frac{F_n}{\omega_0^2 n^2}$

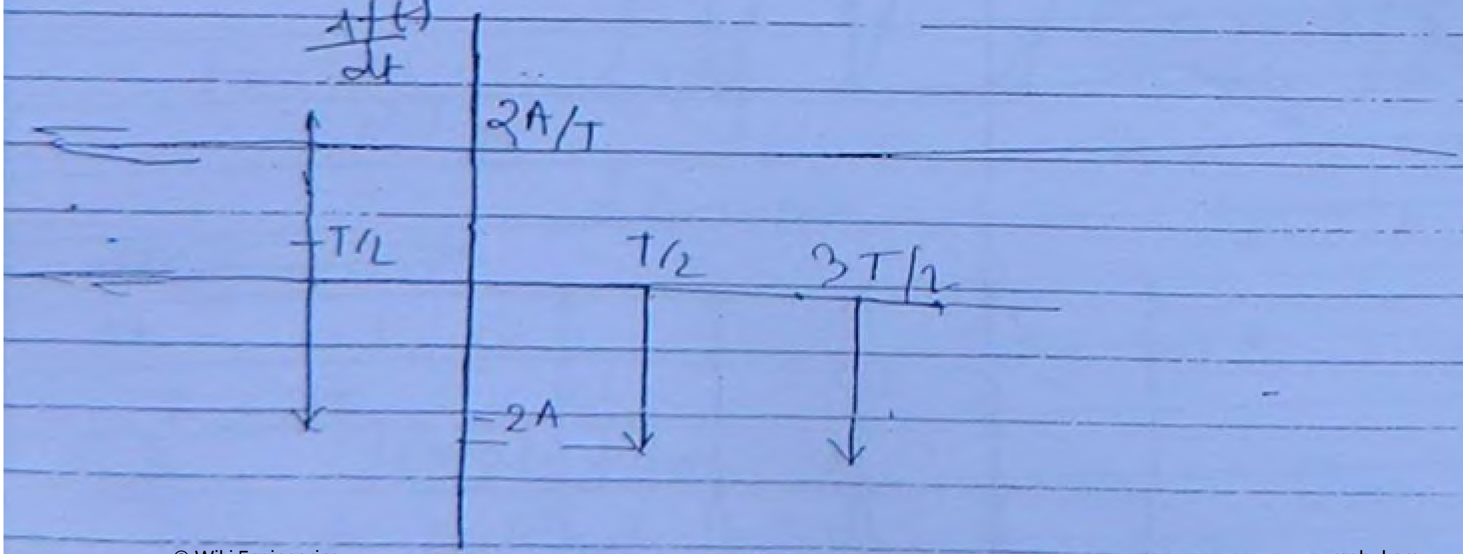
$F_n\left[1 - \frac{1}{\omega_0^2 n^2}\right] = -\frac{2}{\pi \omega_0^2 n^2}$

$\boxed{F_n = \frac{2}{\pi(1-\omega_0^2 n^2)}} = \frac{2}{\pi(1-4n^2)}$

even & real

because ~~signal~~ i f(t) is real
& even





8σ

$\left| \frac{2A}{T} e^{-jn\omega_0 t} \right.$

for $\frac{df(t)}{dt}$

$C_n = \frac{2A}{T} - \frac{2A}{T} e^{jn\omega_0 \cdot T/2}$ ⑺⑺

or

~~$C_n = -\frac{2A}{T} \cdot \frac{2A}{T} e^{-jn\omega_0 T/2}$~~

$C_n' = \text{of} \left(\frac{df}{dt} - \frac{2A}{T} \right) \rightarrow$ only effect C_0 coefficien

$-\frac{2A}{T} e^{jn\omega_0 T/2} = \frac{f[n]}{} \, jn\omega_0 \quad \{ n \neq 0 \}$

$f_n \quad f[n) = -\left(\frac{2A}{T jn\omega_0} e^{-jn\pi} \right) \rightarrow \frac{2A}{jn2\pi} \left[e^{-jn\pi} \right]$

$f_0 \quad f[0] = -\frac{2A}{T} + \frac{2A}{T} = 0$

$= -\frac{A}{jn\pi} \left[(-1)^n \right]$

$f[n] = (-1)^{n+1} \frac{A}{jn\pi}$

$n \neq 0$

$f_0 = 0$

image and odd

Q. A signal $f(t)$ is expressed as a some of sinusoidal signal as shown below

$f(t) = 2 + 3\cos\left(\frac{5\pi}{6} t + \pi/3\right) + 4\sin\left(\frac{6\pi}{5} t + \pi/4\right)$

$+ 8\cos(\pi/7 \, t)$

$= 2 + \frac{3}{2}\left[e^{j(5\pi/6 t + \pi/3)} + e^{-j\frac{5\pi}{6}} \right.$

$\frac{5\pi}{6}, \frac{6\pi}{5}, \pi/7$

Overall

$+ 4 \frac{2}{j} \left[e^{j(\frac{6\pi}{5} t + \pi/4)} - e^{-j\frac{6\pi}{5}} \right.$

$\omega_0 = \frac{\pi}{210}$, $T = \frac{2\pi}{\omega_0}$ 420

$+ \frac{3}{2} \left[\cos e^{j\pi/7 t} - e^{-j\pi/7} \right.$

$$f(t) = r_0 + \sum_{n=1}^{\infty} r_n \cos(n\omega_0 t - \phi)$$

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$$f(t) = 2 + \frac{3}{2} e^{j\pi/3} e^{j\frac{\pi}{210}\times\frac{5}{6}\times\overset{35}{210} t} + \frac{3}{2} e^{-j\pi/3} \cdot e^{-j\pi/210 \frac{5}{6}\times 210 t}$$

$$+ \frac{2}{j} e^{j\pi/4} e^{j\pi/210\times 6/5\times\overset{42}{210} t} - \frac{2}{j} e^{-j\pi/4} e^{j\pi/210\times\frac{6}{5}\times 210 t}$$

$$+ \frac{3}{2} e^{j\pi/210\times\overset{30}{\frac{210}{7}} t} + \frac{3}{2} e^{-j\frac{\pi}{210}\times\frac{210}{7} t}.$$

ac terms, ±175th, ±252th, ±30th harmonics.

175th, 252th, 30th harmonics.

Q. Is the following signal f(t) defined as

$$f(t) = (\cos 2\pi t + \cos 7\pi t)^2$$

$$= \cos^2 2\pi t + \cos^2 7\pi t + 2\cos 2\pi t \cos 7\pi t$$

$$= \frac{1}{2}\left[~~2\cos~~ 1 + \cos 4\pi t + 1 + \cos 14\pi t\right]$$

$$+ 2\left[\cos 5\pi t + \cos 9\pi t\right]$$

$\omega \rightarrow 4\pi, 14\pi, 5\pi, 9\pi \rightarrow$ H.C.F of all $\omega \rightarrow \pi$

π

ac terms are 4th, 14th, 5th, 9th harm

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$P_t = a_0^2 + \sum_{n=1}^{\infty} \left(\frac{a_n^2}{2} + \frac{b_n^2}{2}\right)$$

$$= a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

$$P_t = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} r_n^2$$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0}$$

$$C_n e^{jn\omega_0 t}$$

$$= |C_n|^2,$$

$$P = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$|C_n|^2 = \frac{a_n^2+b_n^2}{4} = |C_{-n}|^2$$

$$|C_n|^2 + |C_{-n}|^2 = 2\,\frac{a_n^2+b_n^2}{2}$$

$$P = C_0^2 + \sum_{n=1}^{\infty} \{|C_n|^2 + |C_{-n}|^2\}$$

$$= C_0^2 + 2\sum_{n=1}^{\infty} |C_n|^2$$

$$= C_0^2 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} |C_n|^2$$

$$\boxed{P = \sum_{n=-\infty}^{\infty} |C_n|^2}$$ Parseval's power relation for periodic power signal

Q. A periodic saw tooth waveform as one considered in previous problem has a period T = 2, find the no. of fourier cofficient to be considered such that considered fourier cofficients 90% of the total power.

$$\sum_{n=-\infty}^{\infty} |C_n|^2 = P_{total}$$

$$P_{total} = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left(\frac{A}{n\pi}\right)^2 = \frac{2A^2}{\pi^2} \cdot \sum_{n=1}^{\infty} \left(1 + \frac{1}{4} + \frac{1}{9} + \cdots\right)$$

$$= \frac{2A^2}{\pi^2} \cdot \sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^2$$

$$P_{total} = \frac{2A^2}{\pi^2}.$$

$$P_{total} = \frac{1}{2} \int_{-1}^{1} (At)^2\, dt = \frac{A^2}{2}\left[\frac{t^3}{3}\right]_{-1}^{1} = \frac{A^2}{2\times 3}\,[2]$$

$$= A^2/3$$

Power in n terms

$$\frac{2A^2}{\pi^2}\sum_{m=1}^{n}\left(\frac{1}{m}\right)^2 = 0.9\times\frac{A^2}{3}$$

$$\sum_{m=1}^{n}\left(\frac{1}{m}\right)^2 = \frac{\overset{0.3}{\cancel{0.9}}}{\cancel{6}\,2}\times\pi^2$$

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25} = \frac{3}{20}\pi^2$$

+1/36

5 terms >90%.

OK.

1.25

9.89×3

1.47

29.67

20

$\boxed{1.38}$

$$\frac{1}{1}+\frac{1}{9}$$

$$\frac{9+}{36}$$

$$)\frac{130}{36}$$

0.04

6 7

1.38 + 0.9$\boxed{0.06}$ +

Q. A periodic signal $f(t) = \sum_{n=0}^{\infty} \cos n\pi\, e^{+jn\pi/100t}$

is this signal is complex valued signal or real signal.

$$C_n = C_{-n}^*$$

$$= \sum_{n=0}^{\infty} \cos n\pi\, e^{j(-n)\pi/100\,t}$$

$$= \sum_{n=-\infty}^{0} \cos n\pi\, \underline{e^{jn\pi/100t}}$$

$$C_n = \cos n\pi$$

for $n \leq 0$

0 otherwise

$$C_{-n} = \cos n\pi$$

$n \geq 0$

C_{-n}

$$C_n = \sum_{n=-\infty}^{0} \cos n\pi$$

$$C_{-n} = \sum_{-n=-\infty}^{0} \cos n\pi$$

$$C_{-n} = \sum_{0}^{\infty} \cos n\pi$$

$$C_{-n}^{*} = \sum_{0}^{\infty}$$

complex valued signal

A discrete time system defined by (81)

$y[n] - y[n-1] = x[n]$ → Recursive system + response is depending on previous respon

find its impulse response

[illegible] — -- δ —

$h[n] - h[n-1] = \delta[n]$

$n=0$ $h[0] = 1 + h[-1]$

~~h[~~

$Y[Z] - z^{-1} Y[Z] = X[Z]$

$$H[Z] = \frac{1}{1-z^{-1}} = u[n] = h[n]$$

IIR system

$y[n] = x[n] + x(n-1)$ non recursive form

$y[n] - y[n-1] = x[n] - x[n-2]$ recursive form

same system

FIR system

find the value of the integral

$$\int_{-\infty}^{\infty}\left[\cos 3t \cdot \delta(3t-3) + e^{-3t}\delta'(2t-2)\right]dt$$

$$= \frac{1}{3}[-1] + \int_{-\infty}^{\infty} e^{-3t}\,\delta_2'[t-1]\,\underline{dt}$$

$$= \cancel{6}\ -\frac{1}{3} + \frac{1}{4}\left[-\frac{d}{dt}e^{-3t}\right]_{t=1}$$

$$= \frac{-1}{3}\ +\frac{1}{4} + 3e^{-3}$$

$$= -\frac{1}{3} + \frac{3}{4}e^{-3} \quad \underline{\text{Ans}}$$

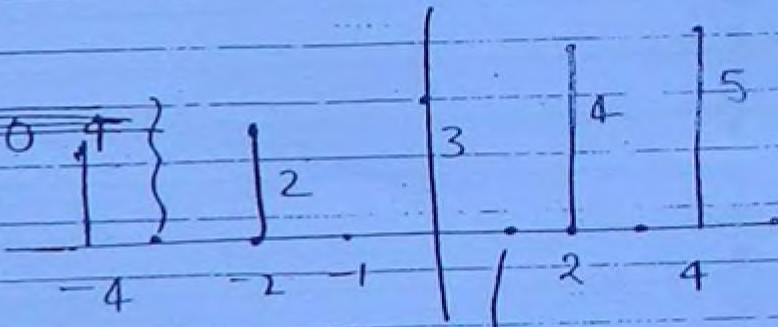
$f[n] \longrightarrow$

$\downarrow \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$f[2n] \longrightarrow \{1\ \underset{\uparrow}{3}\ 5\}$

$\downarrow$
decimation

$f[0\ 1/2\ n]$ ~~$\{2\ 0\ 3\ 0\ 4\}$~~ $\{\ \uparrow\ \}$



we can also ~~fed~~ fill these samples by 1 called unit interpolation.

& fill with zero called zero interpolation

$\delta'(-t) \longrightarrow -1\ \delta'(t) \rightarrow$ add function.

* all even derivative of $\delta(t)$ are even functions of time and all odd derivative of $\delta(t)$ are odd function of time.

$$\frac{d}{dt}\left[f(t)\cdot\delta(t-a)\right] = \frac{d}{dt}\left[f(a)\,\delta(t-a)\right]$$

$$f'(t)\,\delta(t-a) + f(t)\cdot\delta'(t-a) = f(a)\cdot\delta'(t-a)$$

$$\boxed{f(t)\,\delta'(t-a) = f(a)\,\delta'(t-a) - f'(t)\,\delta(t-a)}$$

$$t\,\delta'(t) = 0 \cdot - 1\,\delta(t-0)$$

$$= -\delta(t)$$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\,dt = f(a)$$

$$\int_{-\infty}^{\infty} f(t)\,\delta'(t-a)\,dt = \int_{-\infty}^{\infty} f(a)\,\delta'(t-a)\,dt - \int_{-\infty}^{\infty} f'(t)\,\delta(t-a)\,dt$$

$$= \int_{-\infty}^{\infty} f(a)\,\delta'(t-a)\,dt - f'(a)$$

$$= f(a)\int_{-\infty}^{\infty} \delta'(t-a)\,dt - f'(a)$$

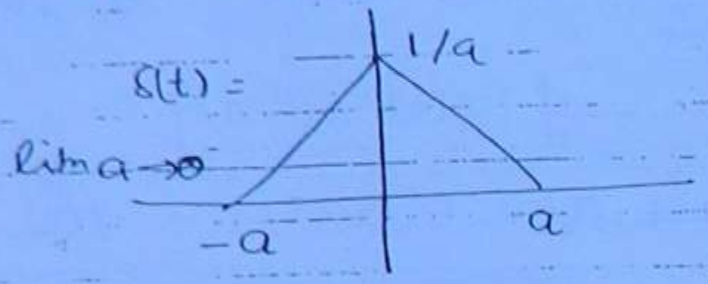
$$= f(a)\left[\delta(t-a)\right]_{-\infty}^{\infty} - f'(a)$$

$$= f(a) \times 0 - f'(a)$$

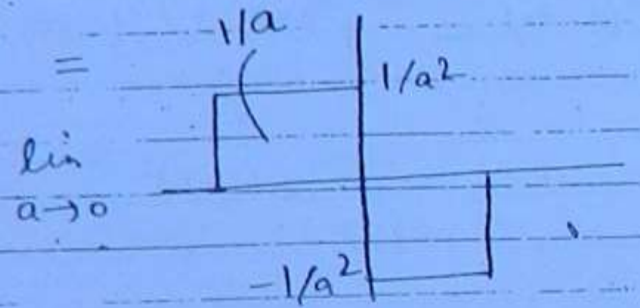
$$= -f'(a)$$

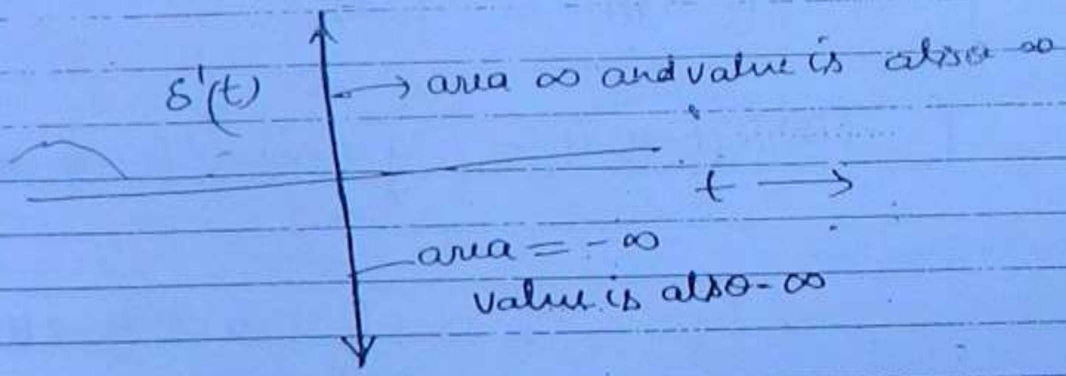
derivative of unit impulse signal

$$\delta'(t) = \frac{d}{dt}\delta(t) = \ldots\ldots$$



$$\underset{a\to 0}{lt}\ \frac{d}{dt}\delta(t) =$$





$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$\frac{d}{dt}\delta(at) = \frac{1}{|a|}\cdot\delta'(t)$$

$$a\,\delta'(at) = \frac{1}{|a|}\,\delta'(t)$$

$$\boxed{\delta'(at) = \frac{1}{a|a|}\,\delta'(t)}$$

Cooley - Tukey

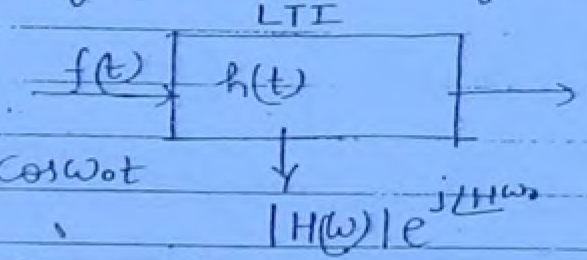
$\downarrow$

FFT (algorithm).

$\frac{N}{2}\log_2 N$ — multiplications.

$N \log_2 N \longrightarrow$ additions.

Group delay and phase delay →



$\longrightarrow |H(\omega_0)| \cos(\omega_0 t + \angle H(\omega_0))$

$\cos\left[\omega_0\left[t + \left(\frac{\angle H\omega_0}{\omega_0}\right)\right]\right]$

Phase delay $\tau_p = -\frac{\phi(\omega)}{\omega}$ (to offer a single frequency) (τ – delay)

$$\phi(\omega) = \angle H(\omega)$$

$\tau_g = -\frac{d\,\phi(\omega)}{d(\omega)}$ (offer to group of frequencies)

Q. Calculate the Group delay and the phase delay of a distortionless transmission system and compare them.

$\angle H(\omega) = -\omega t_0$

$\therefore$ Phase delay $= t_0$

group delay $= t_0$

$$\begin{tikzcd} x[n] \otimes_{2N} h[n] \arrow[r] \arrow[d, Rightarrow] & X[K]\,H[K] \arrow[d, "IDFT"] \\ ?\ x[n] \otimes h[n] & y(n) \text{ of the LTI system} \end{tikzcd}$$

$$\sum |x[n]|^2 = \frac{1}{N} \sum_{K=0}^{N-1} |X[K]|^2$$ Parseval's theorem

Q. DFT of a real valued signal $x[n]$ is

$\{10, 2+3j, A, 3-2j, -4, B, 1+j, C\}$

find the energy of the signal

$= \{10, 2+3j, 1-j, 3-2j, -4, 3+2j, 1+j, 2-3j\}$.

$$E = \frac{1}{8} \sum_{K=0}^{N-1} |X[K]|^2$$

$$= \frac{1}{8}\left[100 + 13 + 2 + 13 + 16 + 13 + 2 + 13\right]$$

$$= \frac{1}{8}\left[\overset{21.5}{172}\right]$$

$$\boxed{E = 21.5 \text{ Joule}}$$

N^2 - multiplication → actually $4N^2$ multiplication

$N(N-1)$ addition → actually $2N(N-1)$ addition.

$x[n] \xleftrightarrow{DFT} X[K]$

$x[[-n]]_N \xleftrightarrow{DFT} X[[-K]]_N \rightarrow X[N-K]$

$x[N-n]$

circular reversal

circular reversal

Q. $x[n] \longleftrightarrow \{10, 4+2j, -4, 4-2j\}$

find DFT of $x[[-n]]_N$

$X[-K] = \{10, 4-2j, -4, 4+2j\}$

$X[[-K]]_N =$ Ans /

$X[-K] = X^*[K]$

$\boxed{X[K] = X^*[-K]}$

even conjugate in nature if $x[n]$ is a real valued signal as in case of other fourier transforms.

all the properties of fourier transform of CT continuous time signal for complex signals are also valid for DFT.

$x[n] \xrightarrow{Np\text{-}DFT} X[K]$

$h[n] \xrightarrow{Np\text{-}DFT} H[K]$

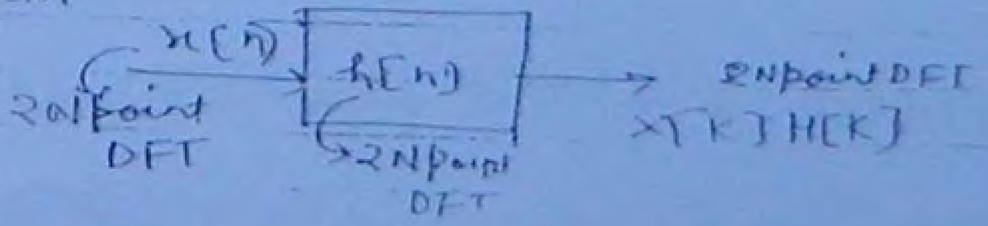
$x[n] \circledast_N h[n] \longleftrightarrow X[K]\,H[K]$

2N zero

$x[n] \xrightarrow{2N\,DFT} X[K]$

$h[n] \xrightarrow{2N\,DFT} H[K]$

for LTI system



⊗.

given two signals f[n] and h(n) each consisting of N-sample values if ~~a~~ we zero pad each of the signal with N-zeros and carry out circuler convolution of order 2N this will be equal to linear convolution of two signals f[n] & h[n].

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Time shifting Property of DFT →

The DFT of a signal $f[n] \Longleftrightarrow \{10, 4-2J, -4, 4+2J\}$

~~x[n]~~ find the DFT of $f[[n-2]]_4$

$$\cancel{X[K]}\, Y(K)\ F[K]\ e^{-JK\omega_0 \cdot 2}$$

$$Y[K]\ F[K]\ e^{-JK\frac{2\pi}{\cancel{4}}\cdot\cancel{2}}$$

$$Y[K] = F[K](-1)^K$$

$$Y[K] = \{10, -4+2J, -4, -4-2J\}$$

$$x[n] \longrightarrow X[K]$$

$$x[n]\, e^{J\frac{2\pi}{N}\cdot K_0 n} \longrightarrow X[[K-K_0]]_N$$

always circuler shift

Q. DFT of a signal $x[n] = \{1, 2, 3, 4\} \longleftrightarrow X[K]$
find the inverse DFT of $\longleftrightarrow X[K-1]$

$$e^{J2\pi/N\cdot 1\cdot n}$$

$$e^{J2\pi/4\cdot n}$$

$$e^{J\pi/2\, n}$$

$$(J)^n$$

$$y[n] = \{1, 2j, -3, -4j\}$$

f[n]	h[n] 4	3	2	1
1	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

linear convolution result of two signal

$y[n] = \{4, 11, 20, 30, 20, 11, 4\}$

add add add

$= \{24, 22, 24, 30\}$

$(1 - e^{-j\theta}) = e^{-j\theta/2}[e^{j\theta/2} - e^{-j\theta/2}]$

devide the result of linear convolution into twoparts such that first part having N sample and then add corresponding parts in order and write as a circular convolution result

N ↓ No of sample in each signal.

Q. circularly convolve the following two signal

$f[n] = \{1, 2\}$

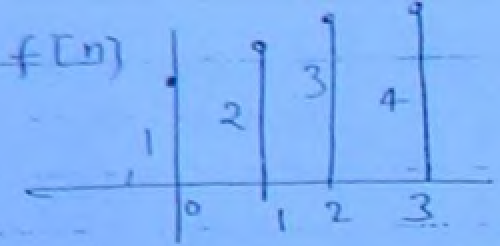
$h[n] = \{2, 1\}$

by zero padding each of signal with two zeros

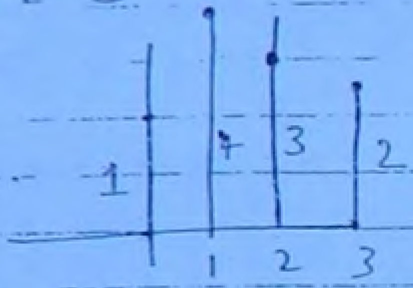
	2	1	0	0
1	2	1	0	0
2	4	2	0	0
0	0	0	0	0
0	0	0	0	0

linear conv^n $[2 \; 5 \; 2 \; 0 \,|\, 0 \; 0 \; 0] \leftarrow f[n] \otimes h[n]$

circular convol → $[2 \; 5 \; 2 \; 0] \leftarrow f[n] \circledast h[n]$



$f[[-n]]_4 \rightarrow$



keep the value at n=0 as it is write ~~other~~ the samples in reverse order.

as 2, 3, 4 ~~until~~ we will write as 4 3 2

n=(1 2 3) at n= (1 2 3)

Q.

Evaluate the circular convolution of 2 signals

$f[n] = \{1,2,3,4\}$

$h[n] = \{4,3,2,1\}$

$\left[f[n] \underset{4}{\otimes} h[n] \right]$ symbol for circular convolution

$$y[n] = \sum_{n=0}^{N-1} f[k]\, h[[n-k]]_4$$

$h[-n] = \{4,1,2,3\}$

$\therefore y[0] = 4+2+6+12 = 24$

$\cancel{4}^{2}(5)(\cancel{8})^{3} = 30$ over $\cancel{6}\;3$

$h[-n+1] = \{3,4,1,2\}$

~~y[1]~~

$\therefore y[1] = 3+8+3+8$
$= 22$

$h[-n+2] = \{2,3,4,1\}$

$y[2] = 2+6+12+4 = 24$

$h[-n+3] = \{1,2,3,4\}$

& $y[3] = 1+4+9+16 = 30$

$y[n] = \{24, 22, 24, 30\}$

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$F[0]$ is always $f[0] + \cdots \quad f[N-1]$

$F[N/2] \quad f[0] - f[1] + f[2] - - - -$

if N is even

$\boxed{F[K] = F^*[N-K]}$ → symmetry satisfied

→ these symmetries are valid till $f[n]$ is a real valued signal.

DFT of a real valued signal is given as

$$\{\underset{0}{10}, \underset{1}{A}, \underset{2}{2+3j}, \underset{3}{B}, \underset{4}{-4}, \underset{5}{3-2j}, \underset{6}{C}, \underset{7}{1+j}\}$$

find A, B, C.

$A = 1-j$

$B = 3+2j$

$C = 2-3j$

$\Big\}$ using $F[K] = F^*[N-K]$

Q. Find the DFT of the following 8-point signal

$f[n] = \{1, - - - 1\}$

$F[0] = 8$

$F[1] = [1 + e^{-j\omega_0} + e^{-2j\omega_0} + - - + e^{-7j\omega_0}$

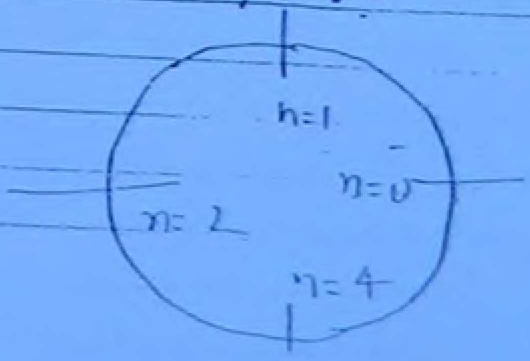
$F[4] = 0$

$$F[1] = 1\left[\frac{1-e^{-8j\omega_0}}{1-e^{-j\omega_0}}\right]$$

$$F[1] = \frac{e^{-4j\omega_0}}{e^{-j\omega_0/2}} \frac{2j \sin 4\omega_0}{2j \sin \omega_0/2}$$

$$= e^{-7j\omega_0/2} \frac{\sin 4\omega_0}{\sin \omega_0/2}$$

Circular shift :→



$f[[n-1]]_4$ $\quad f[N-n]$

shifting 1 unit but not linear bcoz it is circular

$f[[-n]]_4$

N-3/4

If N-sample values are picked up uniformly from DTFT of a discrete time signal in it's unique period of 2π, we get the discrete fourier transform

(92)

Evaluate DFT of $f[n] = \{1, 2, 3, 4\}$

$$F[K] = \sum_{N=0}^{3} f[n]\, e^{-jK\omega_0 n} \qquad W_N = e^{-j\omega_0} \rightarrow \text{twiddle factor}$$

$$\sum_{N=0}^{3} f[n]\, W_N^{Kn}$$

$$F[0] = 10$$

$$F[1] = \sum_{N=0}^{3} f[n]\, e^{-j\omega_0 n}$$

$$= [1 + 2e^{-j\omega_0} + 3e^{-2j\omega_0} + 4e^{-3j\omega_0}]$$

$$= [1 + 2e^{-j\pi/2} + 3e^{-j\pi} + 4e^{-3j\pi/2}]$$

$$= [-2 - 2j + 4j]$$

$$F[1] = -2 + 2j$$

$$F[2] = \sum_{N=0}^{3} f[n]\, e^{-2j\omega_0 n}$$

$$= [1 + 2e^{-2j\omega_0} + 3e^{-4j\omega_0} + 4e^{-6j\omega_0}]$$

$$= [1 + (-2) + 3 - 4]$$

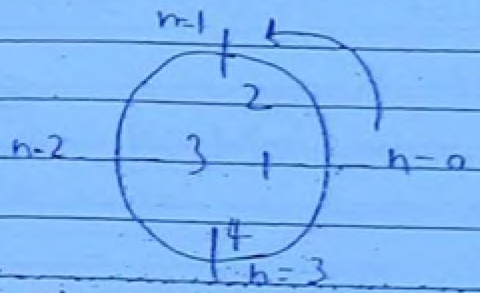
$$F[2] = -2$$

$$F[3] = [1 + 2e^{-3j\omega_0} + 3e^{-6j\omega_0} + 4e^{-9j\omega_0}]$$

$$= [1 + 2j - 3 - 4j]$$

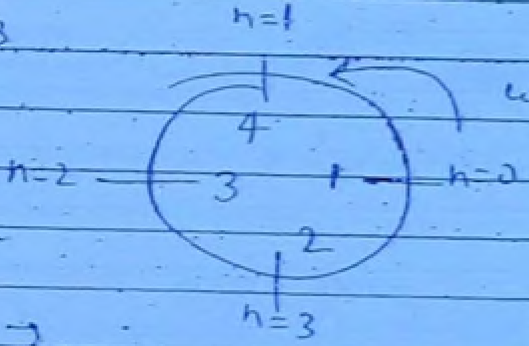
$$= -2 - 2j$$

$$f[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$$



representation → shifting will be done on this

to find $f[(-n)]_4$



write sample in clockwise values.

circular convolution →

$f[n] \rightarrow (0 \rightarrow 3)$ $h[n] \rightarrow (0 \rightarrow 3)$

$y[n] = h[n] \circledast f[n] \longrightarrow 0 \rightarrow 6$ · linear convolution

$= 6$

$y[n] = f[n] \otimes_4 h[n] - \{ \overset{n=0}{} \quad \overset{n=3}{} \}$

↓

circular convolution of order 4

* C_K is defined as discrete time fourier series coefficient of signal f[n], and it is also discrete and periodic with the same period as that of f[n].

$$\text{i.e. } \because \boxed{C_K = C_{K\pm N}} = C_{K\pm mN} \qquad (94)$$

m is an intger

D.F.T.: Discrete Fourier transform →

$$f[n] = \sum_{K=0}^{N-1} C_K e^{jK(\pi\times 2/N)n} \qquad -\infty < n < \infty$$

$$C_K = \frac{1}{N}\sum_{n=0}^{N-1} f[n]\, e^{-jK(2\pi/N)n} \qquad K \rightarrow -\infty \text{ to } \infty$$

discrete time F.T. pairs

D.F.T.

$$F[K] = \sum_{K=0}^{N-1} f[n]\, e^{-jK(2\pi/N)n} \quad \text{for } K = 0 \text{ to } N-1$$

$0 \leq K \leq N-1$

$$\boxed{F[K] = NC_K}$$

$$f[n] = \sum_{K=0}^{N-1} \frac{1}{N} F[K]\, e^{jK(2\pi/N)\cdot n}$$

$$\boxed{f[n] = \frac{1}{N}\sum_{K=0}^{N-1} F[K]\, e^{jK(2\pi/N)\cdot n}}$$

$0 \leq n \leq N-1$

discrete F.T. pairs

- Shifting is circular and

$f[[n-2]]_4$ → order

no of samples taken i.e from 0 to 3

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$$\frac{df(t)}{dt} \leftrightarrow \underline{j\omega} F(\omega)$$

$$f[n] \leftrightarrow F(\omega)$$

$$\boxed{f[n] - f[n-1] \leftrightarrow \boxed{(1-e^{-j\omega})} F(\omega)}$$

$$u[n] \leftrightarrow \frac{1}{(1-e^{-j\omega})} + \pi \delta[\omega]$$

$$-\pi \ \omega \leq \pi$$

$$\leftrightarrow \text{or} \quad \frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi \delta(\omega - 2\pi K$$

$$-\infty < \omega < \infty$$

$$\int_{-\infty}^{t} f(t)\,dt \leftrightarrow \frac{F(\omega)}{j\omega} + \pi\delta(\omega) \cdot F(0)$$

$$f[n] \leftrightarrow F(\omega)$$

$$\sum_{K=-\infty}^{n} f[K] \leftrightarrow \frac{F(\omega)}{(1-e^{-j\omega})} + \pi\delta(\omega) F(0)$$

⊛ Duality Property of F.T. :

× D.T.F.T. and continuous time fourier series forms dual of each other.

× Discreteness in one domain leads to periodicity in the second domain and vice versa.

fourier Series for discrete ~~time~~ signals →

$$f[n] = \sum_{K=} C_K e^{jK\omega_0 n} = \sum_{K} C_K e^{jK\frac{2\pi}{N} n}$$

periodic N

K = ↓ over period N

$$\boxed{C_K = \frac{1}{N} \sum_{n} f[n] e^{-jK\omega_0 n}}$$

n ↓ over period N

we can use the properties of F.T. (continuous or discrete) only if dirchlet condition must be satisfied.

[illegible][n] =

$$E = \frac{1}{2\pi} \int_{\pi}^{\pi} |F(\omega)|^2 \, d\omega$$

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$$E = \frac{1}{2\pi} \int_{-2}^{2} (1)^2 \, d\omega$$

$$\boxed{E = \frac{4}{2\pi} = \frac{2}{\pi}}$$

⊛ Convolution property of D.T.F.T.

$$f(n) \circledast h[n] \longleftrightarrow F(\omega) \, H(\omega)$$

Find the D.D.T.F.T of ~~F(ω)~~ Y(ω) $= \dfrac{1}{(1-ae^{-j\omega})^2}$

$$a^n u[n] \circledast a^n u[n]$$

$$f[n] = \sum_{K=-\infty}^{\infty} f[K] \, f(n-K)$$

$$= \sum_{K=-\infty}^{\infty} a^K u[K] \, a^{n-K} u[n-K]$$

$$= \sum_{0}^{\infty} a^K \, a^{n-K} \, u[n-K]$$

$$= a^n \cdot \sum_{K=0}^{\infty} u[n-K]$$

$n-K \geq 0$

$$= a^n \cdot \sum_{K=0}^{n} \cancel{u[n-K]} \, 1 \quad \text{for } n \geq 0 \quad K \leq$$

$0 \quad n < 0$

$$f(n) = (n+1) a^n \, u[n]$$

$$f[n] = (n+1) \, a^n \, u[n] \quad \text{Ans}$$

$$f(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \quad 2\omega_0$$

$$f(0) = \frac{\omega_0}{\pi}$$

$$f[n] = \frac{\sin \omega_0 n}{\pi n} \quad \text{for } n \neq 0$$

$$= \frac{\omega_0}{\pi} \quad \text{for } n = 0$$

Q. Two signal f[n] & h[n] are convolved to get the signal ~~6~~ y[n], the no of nonzero samples in f[n] is 5 and no. of nonzero samples in h[n] is 3 the maximum possible sample values of f(n) & h(n) l, & K respectively if D.T.F.T of y[n] is Y(ω) what is maxim possible value of Y(0)

~~5l~~

$$\boxed{Y[0] = 5l \cdot 3K}$$

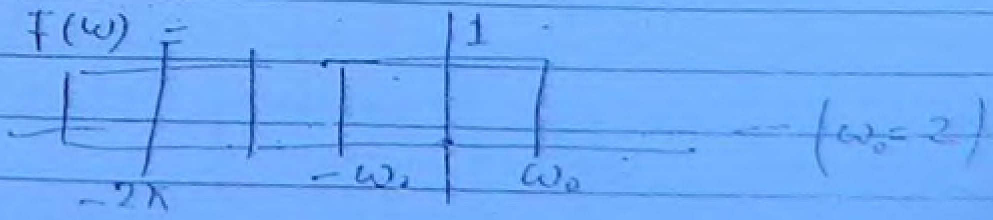
$$\boxed{Y(0) = 15lK}$$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} y[n]\, e^{-j\omega n}$$

$$Y(0) = \sum_{n=-\infty}^{\infty} y[n]$$

Find the energy of the signal $f[n] = \frac{\sin 2n}{\pi n} \quad n \neq 0$

$$\frac{2}{\pi} \quad n = 0$$



$$E = \sum_{n=-\infty}^{\infty} f^2[n] = \frac{4}{\pi^2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{\sin^2 2n}{\pi^2 n^2}$$

Q. a D.T.F.T. of a signal $f[n] \leftrightarrow 2 - 4e^{-j\omega}$
Find the D.T.F.T. $nf[n-2]$

$$f[n-2] \longrightarrow \cancel{2}\ e^{-j2\omega}\left[2\cancel{0} - 4e^{-j\omega}\right]$$

$$nf[n-2] \longleftrightarrow j\left[\cancel{-4je^{-j2\omega}} + 4je^{-j\omega}\right]$$

$$\longleftrightarrow j\left[-4je^{-2j\omega} + 12je^{-3j\omega}\right]$$

$$\longleftrightarrow \left[4e^{-2j\omega} - 12e^{-3j\omega}\right]$$

$$\longleftrightarrow 4e^{-2j\omega}\left[1 - 3e^{-j\omega}\right]$$

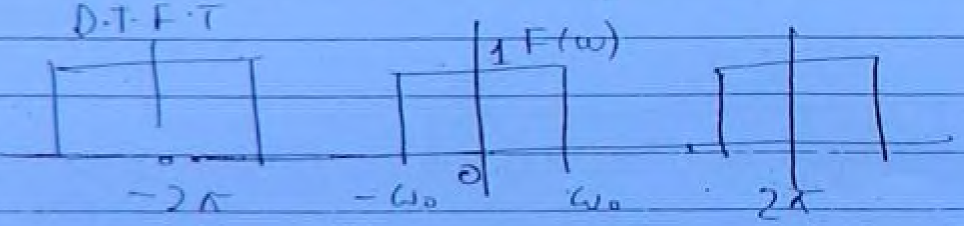
$$\longleftrightarrow$$

$f[n] \longleftrightarrow F(\omega)$

$$\boxed{\sum_{n=-\infty}^{\infty} f[n] = F(0)}$$

$$\boxed{\int_{2\pi} F(\omega)\,d\omega = 2\pi f[0]}$$

Find the inverse D.T.F.T of for following



$$f[n] = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} 1\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_0}^{\omega_0}$$

$$= \frac{1}{2\pi jn}\left[2j \sin\omega_0 n\right]$$

$$\boxed{f[n] = \frac{1}{\pi n}\sin\omega_0 n = \frac{\omega_0}{\pi} Sa[\omega_0 n]}$$

$\sin \omega_0 n \longleftrightarrow \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$

$\pi \leq \omega \leq \pi$

(99)

or $\frac{\pi}{j}\sum_{k=\infty}^{\infty} \delta(\omega-\omega_0-2\pi k) + \delta(\omega+\omega_0-2\pi k$

$-\infty \leq \omega < \infty$

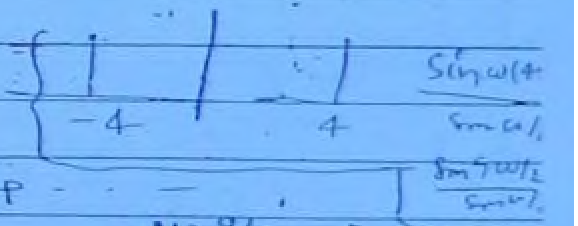
6 $f[n] \longleftrightarrow F(\omega)$ i

$\boxed{F(n-n_0) \longleftrightarrow e^{-j\omega n_0} F(\omega)}$

$\boxed{e^{j\omega_0 n} f[n] \longleftrightarrow F(\omega-\omega_0)}$

Find DTFT of $f[n] = u[n] - u[n-9]$

$$= \sum_{n=0}^{9} f(n) e^{-j\omega n}$$



$$= ~~1+e~~ \quad 1+e^{-j\omega} + \dots$$

$$= \frac{1-e^{-j\omega 9}}{1-e^{-j\omega}} = \frac{e^{-j\omega 9/2}}{e^{-j\omega/2}} \frac{2j\sin \omega 9/2}{2j\sin \omega/2}$$

$$= e^{-j\omega \cdot 4} \frac{\sin 9\omega/2}{\sin \omega/2}$$

$$F[\omega] \Rightarrow e^{-j4\omega} \frac{\sin 9\omega/2}{\sin \omega/2}$$ Ans

If D.T.F.T of $f[n] \leftrightarrow F(\omega)$, find I.D.T.F.T. of $F(\omega-\pi)$

$e^{j\pi n} f[n]$ Ans

$(-1)^{\cancel{n}} f[n] \leftrightarrow F(\omega-\pi)$

$(-1)^n \cdot e\, f[n] \longleftrightarrow F(\omega-\pi)$.

$nf[n] \longleftrightarrow j\frac{dF(\omega)}{d\omega}$

find D.T.F.T. of $na^n u[n] \longleftrightarrow j\frac{d}{d\omega}\frac{1}{1-ae^{-j\omega}}$

$|a|<1$ $\quad = -ae^{-j\omega}$

$$\boxed{f[n] = \frac{1}{2\pi}\int_{2\pi} F(\omega)e^{j\omega n}\,d\omega}$$

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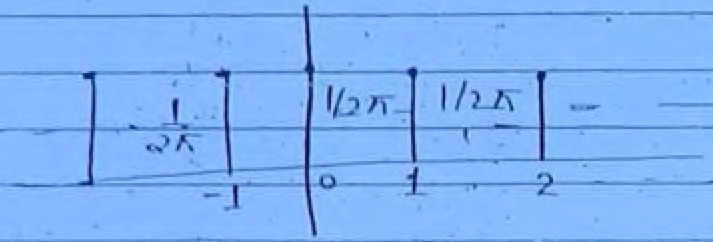
(8). $F(\omega) = \delta(\omega) \xrightarrow{-\pi \le \omega \le \pi}$ or $\rightarrow$

$$F(\omega) = \sum_{-k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

$-\infty < \omega \le \infty$

$$f[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \delta(\omega)\, e^{j\omega n}\,d\omega$$

$$\boxed{f[n] = \frac{1}{2\pi}}$$



$$\frac{1}{2\pi} \longleftrightarrow \delta[\omega] \qquad -\pi \le \omega \le \pi$$

$$1 \longleftrightarrow 2\pi\delta[\omega]$$

$$1 \longleftrightarrow 2\pi \sum_{k=-\infty}^{\infty} \delta[\omega - 2\pi k] \qquad -\infty \le \omega \le \infty.$$

9. D.T.F.T. of $F(\omega) = 2\pi\,\delta(\omega - \omega_0)$

$$= f[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\,\delta(\omega - \omega_0)\, e^{+j\omega n}\,d\omega$$

$\omega - \omega_0 = W$

$$= \frac{1}{2\pi}\,1\int_{(-\pi-\omega_0)}^{(\pi-\omega_0)} \delta(W)\,e^{j(\omega_0 + W)n}\,dW$$

$$f[n] = e^{+j\omega_0 n} \cdot 1$$

$$e^{+j\omega_0 n} \longleftrightarrow 2\pi\,\delta(\omega - \omega_0) \,/\, -\pi \le \omega \le \pi$$

$$e^{j\omega_0 n} \longleftrightarrow 2\pi \sum_{k=-\infty} \delta(\omega - \omega_0 - 2\pi k) \qquad -\infty < \omega < \infty$$

$$\cos \omega_0 n \longleftrightarrow \pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]$$

$-\pi \le \omega \le \pi$

$$\text{or } \pi \sum^{\infty} \delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k).$$

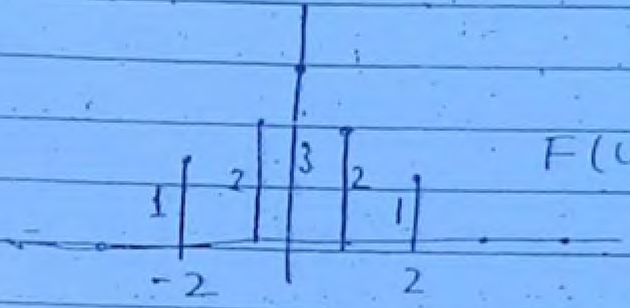
$-\infty$

$$F(\omega) = e^{j\omega N} \cdot e^{-j\omega(2N+1)/2} \frac{\left[e^{j\omega(2N+1)/2} - e^{-j\omega(2N+1)/2} \right.}{e^{-j\omega/2}\left[e^{j\omega/2} - e^{-j\omega/2} \right]}$$

(10).

$$= \cancel{e^{-j\omega N}} \frac{\cancel{2} \sin \omega\frac{(2N+1)}{2}}{\cancel{2j} \sin \omega/2}$$

$$\boxed{F(\omega) = \frac{\sin \omega(N+1/2)}{\sin \omega/2}}$$

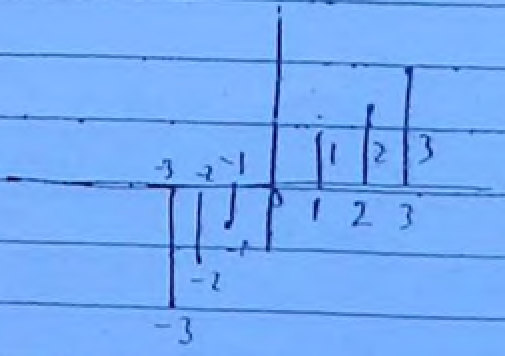


$$F(\omega) = \sum_{n=-2}^{2} f(n) e^{-j\omega n}$$

$$= 3 + 2e^{-j\omega} + 2e^{j\omega} + 1e^{-j\omega \times 2} + 1e^{2j\omega}$$

$$= 3 + 2 2\cos\omega + 2\cos 2\omega$$

$$\boxed{F[\omega] = 3 + 4\cos\omega + 2\cos 2\omega}$$



$$F[\omega] - 2j\sin\omega + 4j\sin 2\omega + 6j\sin 3\omega$$

Ans

D.T.F.T is a continuous function of ω repeating with a period of 2π i.e. we generally consider D.T.F.T in an interval of width of 2π extending from −π to π for all analytical (mathematical) purposes.

$f[n] = a^n u[n]$

$$a^n u[n] \Leftrightarrow \left[\frac{1}{1 - ae^{-j\omega}}\right] \qquad |a|<1$$

(102)

$\delta[n] \Leftrightarrow 1$

$$-a^n u[-n-1] \longleftrightarrow \frac{1}{1-ae^{-j\omega}} \qquad |a|>1$$

$$a^{|n|} = a^n u[n] + a^{-n} u[-n-1]$$

$$a^{|n|} \Leftrightarrow \frac{1}{1-ae^{-j\omega}} + \frac{1}{\left(1-\frac{1}{a}e^{-j\omega}\right)} \qquad |a|<1$$

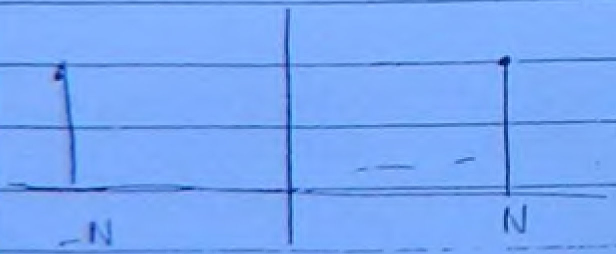
Am

$$= \frac{a-e^{-j\omega} - (1 + ae^{-j\omega})a}{(1-ae^{-j\omega})(a-e^{-j\omega})} \qquad = \left|\frac{1}{a}\right| > 1 \qquad (|a|<1)$$

$$= -\frac{e^{-j\omega}(1-a^2)}{(e^{j\omega}-a)(a-e^{-j\omega})}$$

$$= \frac{1-a^2}{(a-e^{j\omega})(a-e^{-j\omega})} = \frac{(1-a^2)}{a^2 - 2a\cos\omega + 1} \qquad \text{Am}$$

$|a|<1$



$$\longleftrightarrow F(\omega) = \sum_{n=-N}^{N} 1 \cdot e^{-j\omega n}$$

$n = m$

$n+N=m$

$$F(\omega) = \sum_{m=0}^{2N} e^{-j\omega(m-N)}$$

$$= e^{j\omega N} \sum_{m=0}^{2N} e^{-j\omega m}$$

$\delta(n-m)$

$\delta(n+m)$

$= z^{-m}, z^{m}$

$$= e^{j\omega N}\left[\frac{\left[1-e^{-j\omega(2N+1)}\right]}{\left(1-e^{-j\omega}\right)}\right]$$

$$F(\omega) = \frac{e^{j\omega N} - e^{-j\omega N - 1}}{1 - e^{-j\omega}}$$

Initial value theorem can be applied any F(z) there is no restriction.

* For application of Final value theorem all the poles of $(z-1)F(z)$ or $(1-z^{-1})F(z)$ must lie inside the unit circle (strictly i.e $|z| \neq 1$)

{ [illegible]

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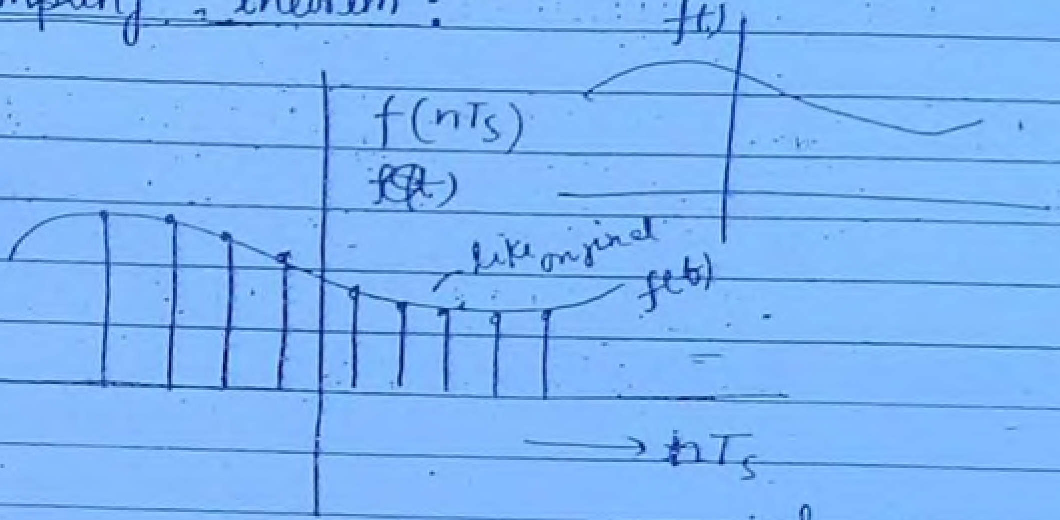
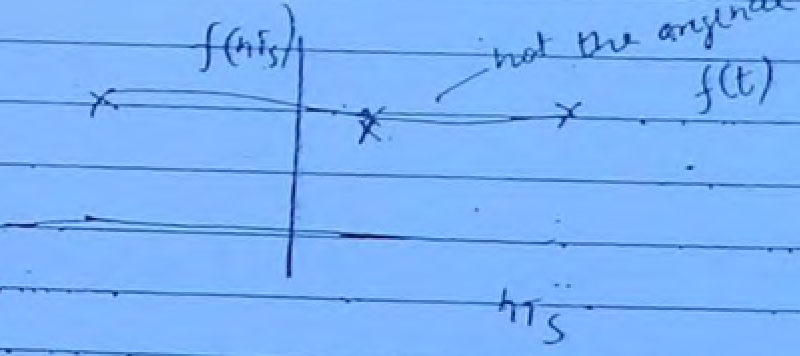
Sampling theorem :





11th NOV 10 :

$$F(\omega) = \sum_{n=-\infty}^{\infty} f[n]\, e^{-j\omega n}$$

or $F(j\omega), F(e^{j\omega})$

discrete time F.T

$F(j\Omega)$

$F(e^{j\Omega})$

omega corresponds to discrete angular frequency

all will for D.T.F.T.

$Y(z) = \frac{1}{(1-2z^{-1})} \cdot \frac{1}{(1-3z^{-1})} = -\frac{1\ 2}{2(1-2z^{-1})} + \frac{3}{(1-3z^{-1})}$

$|z|>3$

$y[n] = -2\ 2^n u[n] + 3 \cdot 3^n u[n]$

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$y[n] = 3^{n+1} u[n] - 2^{n+1} u[n]$

Q. Property

$f[n] \longrightarrow F(z)$

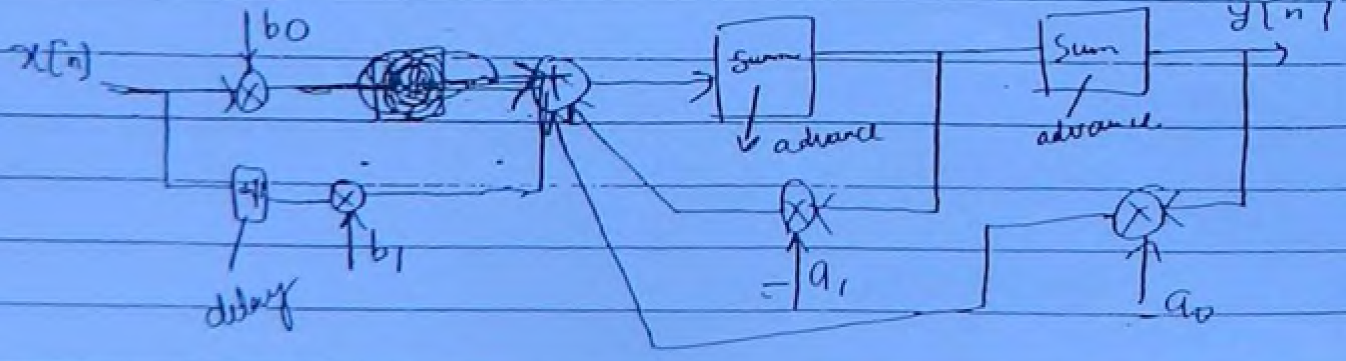
$f[n] - f[n-1] \longrightarrow (1-z^{-1})\,F(z)$, ROC remains same ~~except~~ only exclude $z=0$ from ROC.

$\sum_{k=-\infty}^{n} f[k] \longleftrightarrow f[n] * u[n] \longleftrightarrow F(z) \cdot U(z)$

ROC intersection of the two

$\longleftrightarrow \frac{1}{(1-z^{-1})} \cdot F(z)$

$a_2 y[n-2] + a_1 y[n-1] + a_0 y[n] = b_1 x[n-1] + b_0 x[n]$



$$\left\{ \begin{array}{l} a^n u[n] \longrightarrow \frac{1}{1-az^{-1}} \\ -a^n u[-n-1] \rightarrow \frac{1}{1-az^{-1}} \end{array} \right\}$$

$$\frac{z}{(z+3)^2} \longrightarrow \frac{1}{3} \cdot \frac{z/3}{\left(\frac{z}{3}+1\right)^2} \longrightarrow \frac{-1}{3} \frac{z/-3}{\left[\left(\frac{z}{-3}\right)-1\right]^2}$$

105

$$\rightarrow \frac{-1}{3}(-3)^n n u[n]$$

$$\left\{ \begin{array}{lll} n u[n] \longrightarrow \frac{z}{(z-1)^2} & - & \text{ROC } |z|>1 \\ a^n n u[n] \longrightarrow \frac{z/a}{(z/a-1)^2} & & \text{ROC } |z|>a \end{array} \right\}$$

$$n^2 u[n] \longrightarrow \frac{z(z+1)}{(z-1)^3}.$$

Q. Z-transform of a signal $f[n]$ is defined as

$F(z) = 2 - 4z^{-1} + 8z^{-2}$

$$f[n] = \{ \underset{\uparrow}{2}, -4, 8 \}$$

$$nf[n] = \{ \underset{\uparrow}{0}, -4, 16 \}$$

$-z\frac{dF(z)}{dz}$

Zwt $nf[n]$

$-8(-2)z^{-2}$

$F(z) = G(z) = -4z^{-1} + 16z^{-2}$ An

Convolution Property of z-transforms ⟶

$$f[n] \longrightarrow F(z)$$

$$h[n] \longrightarrow H(z)$$

$$f[n] \otimes h[n] \longrightarrow F(z) \cdot H(z)$$

Q. Find the response of a discrete time system with impulse response $h[n] = 2^n u[n]$

$f[n] = 3^n u(n)$

Shifting

$f[n] \longrightarrow F[z]$

$f[n-n_0] \longrightarrow z^{-n_0}F[z]$ ROC remains same the only z=0 or ∞ ~~are~~ can be excluded.

$\cos\omega_0 n\, u[n] \longrightarrow \dfrac{2z^2 - 2z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}$ $|z|>1$

$\sin\omega_0 n\, u[n] \longrightarrow \dfrac{z\sin\omega_0}{z^2 - 2z\cos\omega_0 + 1}$ $|z|>1$

$-\cos\omega_0 n\, u[-n-1] \longrightarrow \dfrac{2z^2 - 2z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}$ $|z|<1$

$-\sin\omega_0 n\, u[-n-1] \longrightarrow \dfrac{z\sin\omega_0}{z^2 - 2z\cos\omega_0 + 1}$ $|z|<1$

$a^n \cos\omega_0 n\, u[n] \longrightarrow \dfrac{2(z/a)^2 - 2z/a\cos\omega_0}{(z/a)^2 - 2z/a\cos\omega_0 + 1}$ $|z|>|a|$

Multiplication by 'n' :

$a^n n\, u[n] \rightarrow \dfrac{z/a}{(\frac{z}{a}-1)^2}$

$n u[n] \rightarrow -z \dfrac{d}{dz}\left(\dfrac{z}{z-1}\right) = -z\left[\dfrac{-z}{(z-1)^2} + \dfrac{1}{z-1}\right] = -z[$

$f[n] \longrightarrow F(z)$

$nf[n] \longrightarrow -z\dfrac{dF(z)}{dz}$ ROC remains same

$\left(\dfrac{z}{z-1}\right)$

Find Inverse Z-T. $F(z) = \dfrac{z}{(z-2)(z+3)^2}$

$\dfrac{1}{z-2i} \quad \dfrac{-1}{(z-2)^2}$

$\dfrac{F(z)}{z} = \dfrac{1}{25(z-2)} - \dfrac{1}{25(z+3)} + \dfrac{-1}{5(z+3)^2}$

$\dfrac{z}{(z+3)^2}$

$\dfrac{3}{(z+3)} \rightarrow -z\left[\dfrac{3}{(z+3)^2} - \dfrac{1}{z+3}\right]$

$F(z) = \dfrac{1}{25}\dfrac{z}{z-2} - \dfrac{1}{25}\dfrac{z}{z+3} - \dfrac{1}{5}\dfrac{z}{(z+3)^2}$

$\dfrac{1}{z+3} = +z\dfrac{1}{(z+3)^2}$

$= \dfrac{1}{25}2^n u[n] - \dfrac{1}{25}(-3)^n u[n] - \dfrac{1}{5}\cancel{n(-3)}$

$+\dfrac{1}{15}n(-3)^n u[n]$

$\dfrac{z}{(z+3)^2}\left(\dfrac{z}{z+3}\right)$

$|z|>3$

when ROC not given consider right

$f[n] = \frac{1}{3}[2]^n u[n] - \frac{1}{3}(-1)^n u[n]$ { Right sided, not stable, Causal, DTFT can't be defin

(10) $|z| > 2$

or $-\frac{1}{3}(2)^n u[-n-1] + \frac{1}{3}(-1)^n u[-n-1]$ { left sid, not stab, not cau, DTFT ca def

$|z| < 1$

or $-\frac{1}{3}(2)^n u[-n-1] - \frac{1}{3}(-1)^n u[n]$

$1 < |z| < 2$ { two sided, DTFT not de, not stable, not causal

1. Find the g.z.T. of above signal if z-transform F(z) is defined for $z = 3+j5$

$|(8)| = |z| = \sqrt{9+25} = \sqrt{34}$

$|z| > 2$ ✓

$f[n] = \frac{1}{3} 2^n u[n] - \frac{1}{3}(-1)^n u[n]$

$|z| > 2$

Linearity:

$a_1 f_1[n] + a_2 f_2[n] \longrightarrow a_1 F_1[z] + a_2 F[z]$

ROC is intersection of individual ROC's

② $a^n f_1[n] \longrightarrow F(z/a)$, ROC $\left|\frac{z}{a}\right|$

$e^{j\omega_0 n} f[n] \longrightarrow F(z/e^{j\omega_0})$ ROC $|z|$ is remains same

because $\left|\frac{z}{e^{j\omega_0}}\right|$

$\frac{|z|}{|e^{j\omega_0}|} = |z|$ ✓

is the DTFT for the previous question is defined
for $|b|<1$
ie if it is defined then it will be stable.

~~DTFT of the fourier~~

(108)

Inverse Z-transform :

$$\boxed{f[n] = \frac{1}{2\pi j} \oint F(z)\, z^{n-1}\, dz}$$

Q:

$$F[z] = \frac{z}{(z^2-z-2)}$$

$$= \frac{z}{(z-2)(z+1)}$$

$$= \frac{2}{3(z-2)} + \frac{1}{3(z+1)}$$

$\Rightarrow$ $= \frac{2}{3}$

$$F[z] = \frac{z^{-1}}{1-z^{-1}-2z^{-2}} = \frac{-z^{-1}}{2z^{-2}+z^{-1}-1}$$

$$= \frac{z^{-1}}{(2z^{-2}+2z^{-1}-z^{-1}-1)}$$

$$= \frac{-z^{-1}}{(2z^{-1}-1)(z^{-1}+1)}$$

$$= \frac{z^{-1}}{(1-2z^{-1})(1+z^{-1})} = \frac{1}{3(1-2z^{-1})} - \frac{1}{3(1+z^{-1}}$$

$\delta[n]\{\underset{\uparrow}{1}\} = 1$ ROC entire z-plane

$f[n] = \{\underset{\uparrow}{1}, 2, 3\}$, $F[z] = 1 + 2z^{-1} + 3z^{-2}$ entire z-plane except $|z| = 0$

$f[n] = \{1, 2, \underset{\uparrow}{3}\}$, $F[z] = 3 + 2z + 1z^{2}$ entire z-plane except $|z| = \infty$

$f[n] = \{1, \underset{\uparrow}{2}, 3\} = z + 2 + 3z^{-1}$ ROC entire z plane except $|z| = 0, \infty$.

* A discrete time signal having finite no. of nonzero sample will have an ROC which is the entire z-plane except possibly at $r = 0$, or $r = \infty$, or both at $r = 0$ and at $r = \infty$.

* If the ROC is to be outward directed, ROC must be extending till $r = \infty$ & including $r = \infty$ circle

Q. $f[n] = b^{|n|}$

$$= b^{n}u[n] + b^{-n}u[-n-1]$$

$$= \frac{z}{z-b} - \frac{z}{z-1/b}$$

~~$b>1$~~ $0<b<1$

$|z| > |b|$ $\quad |z| < |1/b|$ $\quad$ if $\rightarrow |b| < 1$

$$\boxed{\text{ROC } |b| < |z| < |1/b|}$$

if $|b| > 1$ then z-transform can't be defined.

* Combination of left sided signal will have always a defined z-transform with an ROC which is inward directed bounded by the circle whose radius is the magnitude of the least pole. (1/10)

Q. $f[n] = (3/4)^n u[n] + (5/6)^n u[n] + \left(\frac{7}{8}\right)^n u[-n-1] + \left(\frac{9}{10}\right)^n u[-n-1]$

$$F(z) = \frac{z}{z-3/4} + \frac{z}{z-5/6} - \frac{z}{z-7/8} - \frac{z}{z-9/10}$$

$$\left(5/6 < |z| < 7/8 \right),$$

$4^{1/3}$ $5^{1/2}$

$\boxed{6^{1/3} \quad 5^{1/2}}$

$(6^2)^{1/6}$ $(5^3)^{1/6}$

$(36)^{1/6}$ $(125)^{1/6}$

* Combination of right sided and left sided signal will have an ROC which is a finite circular strip bounded on the lower side by a circle whose radius is the magnitude of the greatest pole of all the right sided signals and on upper side by a circle whose radius is the magnitude of the least pole of all the left sided signal.

find the z-transform of signal

$$f[n] = 2^n u[n] + 5^n u[n] - 2^n u[-n-1] - 5^n u[-n-1]$$

$$\left\{ \frac{z}{z-2} + \frac{z}{z-5} + \frac{z}{z-2} + \frac{z}{z-5} \right\}$$

$|z| > 5$ { Simultaneously not possible } { not defined.

$|z| < 2$ {

of the impulse response of such a stable & causal system must lie inside the unit circle

* For a discrete time signal f[n], fourier transform can be defined as $F(\omega) = \sum_{n=-\infty}^{\infty} f[n] e^{-j\omega n}$, this is defined as discrete time Fourier transform or DTFT in short.

⊛ DTFT of a distrete time signal F[n] is same as z-transform, with $r = 1$.

⊛ So in the z-transform expression if we substitute i.e. $z = re^{j\omega}$ then $r = 1$ it will result in discrete time Fourier transform. i.e. If we substitute $z = e^{j\omega}$ in z-transform expression we get the DTFT provided z-transform is defined for $r = 1$ i.e. ROC of the z-transform is including the unit circle

⊛ DTFT is z-transform evaluated along the unit circle

$$\left(3/2\right)^n u[n] \longrightarrow \frac{z}{z-3/2} \qquad |z| > 3/2$$

$$\left(\frac{3}{2}\right)^n u[n] + \left(\frac{5}{2}\right)^n u[n] \longrightarrow \frac{z}{z-3/2} + \frac{z}{z-5/2} \qquad \overset{ROC}{|z| > 5/2}$$

* Combination of right sided ~~sign~~ discrete time signal will always have a defined z-transform with an ROC which is outward directed bounded by a circle whose radius is the magnitude of the highest pole.

$$-2^n u[-n-1] \longrightarrow \frac{z}{z-2} \qquad |z| < 2$$

$$-5^n u[-n-1] - 2^n u[-n-1] \longrightarrow \frac{z}{z-5} + \frac{z}{z-2}$$

S plane will be map to all the circles having radius greater than 1, and jω axis of S plane will be map to a circle of unit radius.

$$\boxed{F[\omega] = \sum_{n=-\infty}^{\infty} f[n] e^{-j\omega n}}$$

DTFT

Discrete time fourier transform

* ROC of z-transform is a set of concentric circles, generally a circular strip.

* ROC of the z-transform can't include any poles but is bounded by the circle whose radius is the magnitude of pole

* ROC of right sided signal is outward directed i.e. outside some circle.

* ROC of left sided signal is inward directed i.e. inside some circle

* If a system with impulse response h[n] is to be stable condition $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$ and for this to be satisfied the region of convergence of the z-transform of the impulse response must be including unit circle.

* A system with impulse response h[n] is causal if h[n] = 0 for n < 0. And in this case the region of convergence of z-transform of impulse response must be outward directed

* If a system with impulse response h[n] is to both stable & causal ROC must be including the unit circle and must be outward directed and ROC can't include any poles.

So for a system with impulse response h(n) to be stable and must all the poles of the z-transform

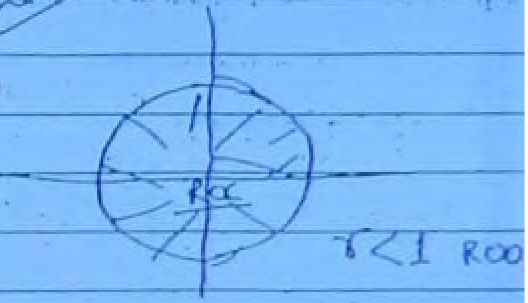
Find z-transform of $f[n] = u[-n-1]$

1/3

$F[z] = z + z^2 + \cdots$

$F[z] = z \frac{1}{(1-z)}$ $|z| < 1$

$F[z] = \frac{-1}{(1-z^{-1})}$ $|z| < 1$ ROC, $r < 1$



$a^n u(n) = \frac{1}{(1-az^{-1})}$ $|az^{-1}| < 1$ $|\frac{z}{a}| > 1$

$|z| > |a|$ $|z| > |a|$

$-a^n u[-n-1] = \frac{1}{(1-az^{-1})}$ $|z| < |a|$ $|\frac{z}{a}| < 1$

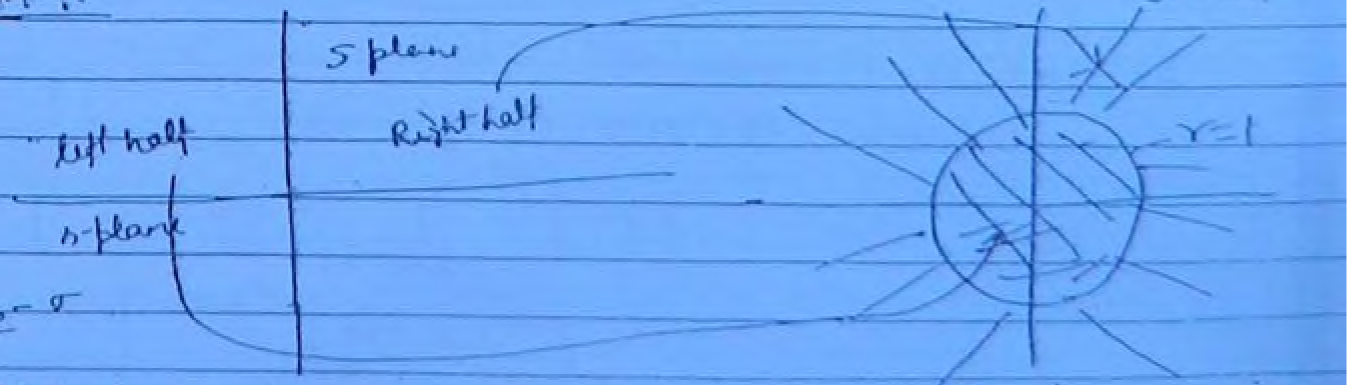
$|z| < |a|$

$f[n] \longrightarrow F(z)$

according to

$a^n f[n] \rightarrow F[z/a]$ ROC $\rightarrow |\frac{z}{a}|$

2nd Nov 10



$r = e^{-\sigma}$

left half of Splane will be map to all circle having Radius [illegible]

For a discrete time signal $f[n]$, z-transform is defined as $F[z] = \sum_{n=-\infty}^{\infty} f[n] z^{-n}$

where z- is complex variable defined as $z = re^{j\omega}$. this can be understood as laplace transform a discrete time signal $f[n]$, the complex variable $s = \sigma + j\omega$ ~~from~~ in rectangular ~~form~~ form is replaced by a complex variable $z = re^{j\omega}$ in polar form. If the z-transform of the signal $f[n]$ is to be defined, the condition is

$$\left|\sum_{-\infty}^{\infty} f[n] z^{-n}\right| < \infty$$

$$\sum_{n=-\infty}^{\infty} |f[n] z^{-n}| < \infty$$

$$\sum_{n=-\infty}^{\infty} |f[n] r^{-n}| < \infty$$

Best on the nature of given discrete time signal $f[n]$ there will be a set of r values for which above conditi is satisfied, this region of r values for which the z-transform of a signal $f[n]$ is defined is called as region of convergen of z-transform and it is specified in terms r or $|z|$ (ROC)

find z-transform of

$$f[n] = \delta[n]$$

$$F(z) = 1$$

[illegible] $1 + z^{-1} + \cdots$

$$\frac{1}{1}$$

$$f[n] = u[n]$$

$$= \quad F[z] = \frac{1}{(1-z^{-1})} \qquad \text{ROC } |z^{-1}| < 1$$

$$\text{ROC} \rightarrow \left\{ |z| > 1 \right|$$

Calculate the final value of $F(S) = \frac{S+2}{(S^2+3S+2)}$

(11B)

$$\lim_{S\to 0} SF(S) = \lim_{S\to 0} \frac{S(S+2)}{(S+2)(S+1)}$$

$$= 0$$

Z-transform:

↳

$$F(Z) = \sum_{n=-\infty}^{\infty} f[n]\, Z^{-n} \qquad \boxed{Z = e^{j\omega}}$$

$$F(Z) = \sum_{n=-\infty}^{\infty} f[n]\, e^{-\sigma n}\, e^{-j\omega n}$$

$S = (\sigma + j\omega)$

$\downarrow \qquad \downarrow$

$Z = r e^{j\omega}$

$r = e^{\sigma}$

$$= \sum_{n=-\infty}^{\infty} f[n]\, r^{-n} (e^{j\omega})^{-n}$$

$$= \sum_{n=-\infty}^{\infty} f[n]\, r^{-n}\, Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} f[n]\, [r e^{j\omega}]^{-n}$$

$$r e^{j\omega} = Z$$

$$F[Z] = \sum_{n=-\infty}^{\infty} f[n]\, Z^{-n}$$

for Z-transform to be defined summation must be converging

$$\left| \sum_{n=-\infty}^{\infty} f[n]\, Z^{-n} \right| < \infty$$

$$\sum_{n=-\infty}^{\infty} \left| f[n]\, Z^{-n} \right| < \infty$$

$$\sum_{n=-\infty}^{\infty} \left| f[n]\, r^{-n} \right| < \infty$$ for a set of r values

whenever result comes $\infty \longrightarrow$ not defined.
$\infty - \infty$ because ∞ is a undefined quantity

Initial value theorem $\longrightarrow$

(116) $\lim \lim_{t \to 0} f(t) = \lim_{S \to \infty} SF(S)$

Initial value theorem $\boxed{f(0^-) = \lim_{S \to \infty} SF(S)}$

Final value theorem $\boxed{\lim_{t \to \infty} f(t) = \lim_{S \to 0} SF(S)}$

$F(S) = \frac{\omega_0}{S^2 + \omega_0^2}$ = Sinusoidal

$\boxed{-1 \le f(t) \le 1}$

(*) Initial value theorem can be applied only if degree of numerator of L.T. is less than the degree of denominator

$$F(S) = \frac{S+2}{S^2+3S+2} = \frac{A}{(S+1)} + \frac{B}{S+2}$$

I.L.T exists

$$F(S) = \frac{S^2+2S+3}{S^2+3S+2} = \frac{S^2}{S^2} 1 + \frac{1-S}{S^2+3S+2}$$

$\delta(t)$

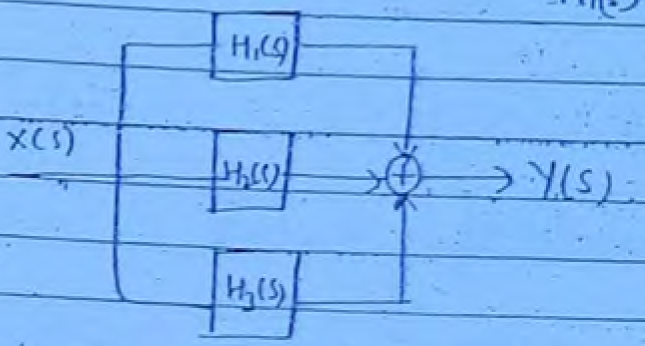
(*) If the degree of numerator is greater than or equal to degree of denominator, we will have derivatives of impulses or impulses themselves in I.L.T whose value is not known at $t=0$ hence initial value theorem can't be applicable.

(*) Final value theorem of L.T. can be applied only if all the poles of S.F(S) lie in left half of the S-plane (Strictly left half i.e. excluding $j\omega$ axis

Parallel Representation.

$$H(S) = \frac{A}{(S+a_n)} + \frac{B}{(S+B)} + \frac{C}{S^2+B^2} + \cdots$$

$$H_1(S) + H_2(S) + H_3(S)$$



(*) $F(S) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$

two sided L.T.

Bilateral L.T.

$F(S) = \int_{0}^{\infty} f(t)\, e^{-st}\, dt$ one sided L.T.

also called as unilateral L.T.

when analysis is started from $t = 0$ ($f(t)$ may exist before $t = 0$)

In unilateral transform ROC does not play important role while it plays an important role in bilateral transform.

(*) Unilateral transform for a signal $f(t)$ is defined as

$F(S) = \int_{0}^{\infty} f(t)\, e^{-st}\, dt$ → also called as one sided

L.T. and unilateral transform for any signal $f(t)$, is unique and hence region of convergence need not be specified to denote the uniqueness of L.T.

$f(t) \longleftrightarrow F(S)$ [$F(S)$ is unilateral transform of $f(t)$]

$f(t-t_0) \cdot u(t-t_0) \longleftrightarrow F(S)\, e^{-St_0}$

$$\frac{df(t)}{dt} \longrightarrow \underbrace{SF(S) - f(0^-)}$$

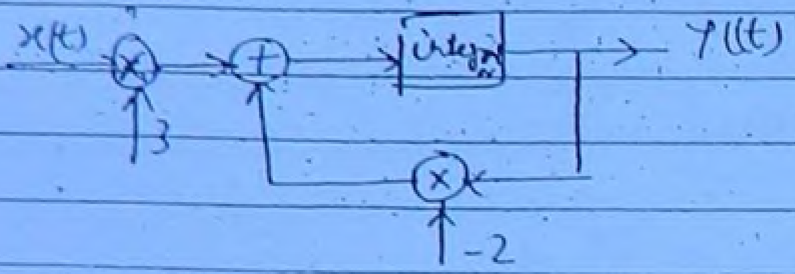
analysis start at $t = 0$

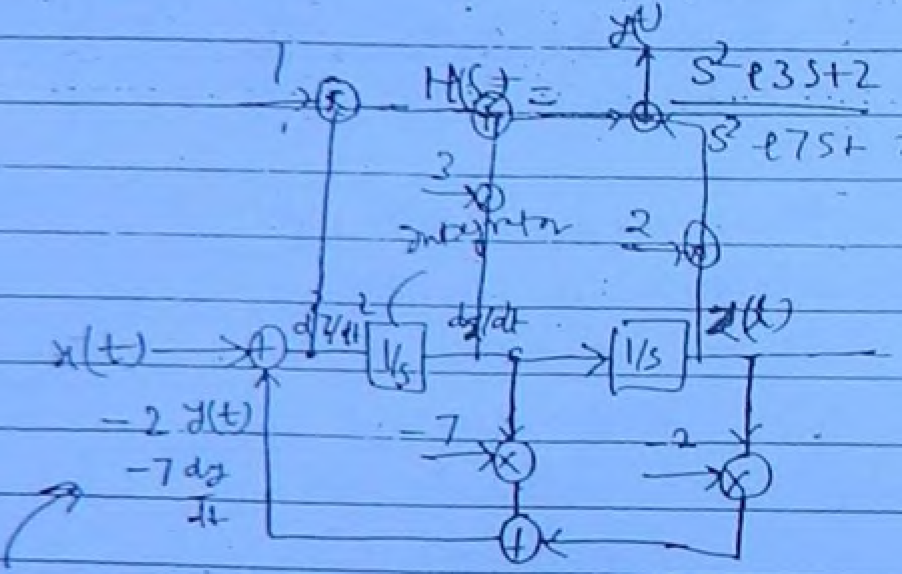
$$\frac{dy}{dt} + 2y(t) = 3x(t)$$

$$\frac{dy}{dt} = -2y(t) + 3x(t)$$

$$H(S) = 3/\underline{S+2}$$

$$\boxed{h(t) = 3e^{-2t}\,\underline{u}(t)}$$





$$H(S) = \frac{S^2+3S+2}{S^2+7S+2} = \frac{1}{S^2+7S+2} \cdot (S^2+3S+2)$$

$H(S)$ $\quad$ $H_2(S)$

$\frac{d^2}{dt^2} \cdots = y(t$

Cascade Representation of LTI systems:-

$f(t) \longleftrightarrow F(S)$

$\frac{d\,f(t)}{dt} \longrightarrow SF(S) - f(0-)$

$\frac{d^2 f(t)}{dt^2} \longrightarrow S^2 F(S)$ ~~$- S f(0-) - f'(0-)$~~

$\int_{-\infty}^{t} f(\tau)\, d\tau \longrightarrow \frac{F(S)}{S}$ ~~[illegible]~~

Q. A system is defined by the following differential eqn

$$\frac{d^2 y}{dt^2} - 2\frac{dy}{dt} + 10\, y(t) = 2\frac{dx}{dt} + 3x(t)$$

Is this linear T.I. system. Yes this is LTI system.

$$H(S) = \frac{2S+3}{(S^2 - 2S + 10)}$$

find h(t) for it to be Stable

$$= \frac{2S+3}{(S-1)^2 + 9}$$

$$H(S) = \frac{2[S-1] + 5}{(S-1)^2 + 9}$$

$$H(S) = \frac{2(S-1)}{(S-1)^2 + 9} + \frac{5}{(S-1)^2 + 9} \qquad \sigma < 1$$

$$h(t) = -2e^{t} \cos 3t\, u(-t) - \frac{5}{3} e^{t} \sin 3t\, u(-t)$$

Ans

$t < 1$ σ

Find impulse response of above system is to be causal

$$= 2e^{t} \cos 3t\, u(t) + 5/3\, e^{t} \sin 3t\, u(t)$$ Ans

$\sigma > 1$

Q. $h(t) = e^{-t} u(t)$, $f(t) = e^{-3t} u(t) + e^{-2t} u(-t)$

$H(S) = \frac{1}{S+1}$, ~~$F(S) = \left[\frac{-1}{S+3} + \frac{1}{S+2}\right]$~~ $\Rightarrow$ (118)

$\sigma > -1$

$F(S) = \left[\frac{1}{S+3} - \frac{1}{S+2}\right]$ $-3 < \sigma < -2$

$Y(S) = H(S) \cdot F(S)$.

~~$-1 < \sigma -2$~~

not defined for any common σ

So $\boxed{y(t) = 0 \text{ for all } t}$.

Convolution Property of L.T. :

$f(t) \longleftrightarrow F(S)$

$h(t) \longleftrightarrow H(S)$

$f(t) * h(t) \longleftrightarrow F(S)\cdot H(S)$

Q. Find the response of an L.T.I system with impulse response $h(t) = e^{2t}[u(t)]$ for i/p $f(t) = e^{3t}u(t)$

$$Y(S) = H(S)\cdot F(S) = \frac{1}{(S-2)(S-3)}$$

$$Y(S) = \frac{1}{S-3} - \frac{1}{S-2}$$

$$y(t) = e^{3t}u(t) - e^{2t}u(t) \quad \sigma > 3$$

$$y(t) = (e^{3t} - e^{2t})\,u(t) \quad \text{Ans}$$

or

$$-e^{3t}u(-t) + e^{2t}u(t)$$

$\sigma < 2$ Ans

Q. Find the response of a system with impulse response $h(t) = e^{-t}u(t)$ for i/p $f(t) = e^{-2|t|}$

$$Y(S) = \frac{1}{(S+1)}\frac{(-4)}{(S^2-4)} \quad -1 < S < 2$$

$$Y(S) = -\frac{4}{(S+1)(S-2)(S+2)} \quad \boxed{-1 < S < 2}$$

$$= \frac{4}{3(S+1)} - \frac{1}{3(S-2)} - \frac{1}{(S+2)}$$

$$y(t) = \frac{4}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t) - e^{-2t}u(t)$$

Ans

e^{-2t} $\frac{1}{S+2} - \frac{1}{}$

$e^{-2t}u(t)$
$+ e^{2t}u(-t)$

$\frac{1}{S+2}$ $\frac{1}{(S-2)}$

$-2 < S < 2$

Find I.L.T. of $F(s) = \log \frac{s^2+a^2}{(s^2+b^2)}$ ⑫⓪

$= \log_e s^2+a^2 - \log_e s^2+b^2$

$$G(s) = \frac{dF(s)}{ds} = \frac{1 \cdot 2s}{s^2+a^2} - \frac{1 \cdot 2s}{s^2+b^2}$$

$g(t) = [2 \cos at\, u(t) - 2 \cos bt\, u(t)]$

$\therefore f(t) = -\cancel{t}\, \cancel{f(t)}\, g(t)/t$

$f(t) = \frac{2\cancel{t}}{t}[\cos bt - \cos at]\, u(t)$ Ans

$t f(t) \longrightarrow -\frac{d}{ds} F(s)$

$t f(t) \longrightarrow -g(t)$

$f(t) = -\frac{g(t)}{t}$

division by t property →

$$\frac{f(t)}{t} \leftrightarrow \int_s^\infty F(s)\, ds$$

Find L.T. $\frac{\sin t}{t} u(t) = Sa[t]$

$$= \int_s^\infty \frac{1}{(s^2+1)} ds = [\tan^{-1} s]_s^\infty$$

$L\left[\frac{\sin t}{t}\right] = \pi/2 - \tan^{-1} s$

$= \cot^{-1} s$

$\theta = \pi/2 - \theta)$

$x = s = \tan(\pi/2 - \theta)$

$s = \cot\theta$

$= \cot^{-1} s$

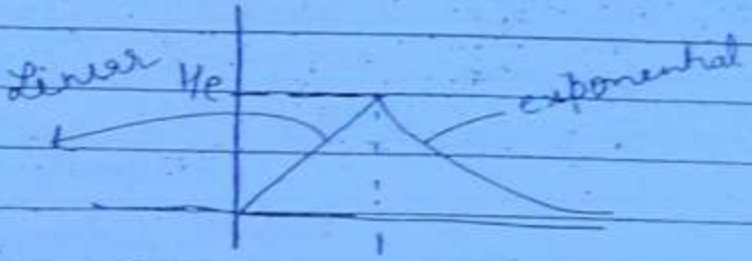
θ

$$\frac{1}{(s-2)(s+3)^2} = \frac{1}{25(s-2)} \quad \frac{1}{25(s+3)} \quad \frac{1}{5(s+3)^2}$$ (121)

$$= \frac{1}{25} e^{2t} u(t) - \frac{1}{25} e^{-3t} u(t) - \frac{1}{5} e^{-3t} t\, u(t)$$ Ans

ROC $(\sigma > 2)$

Q. Find L.T. of the following signal



$$f(t) = \frac{1}{e} t [u(t) - u(t-1)] + ~~e^{-t} [u(t) - u(t}~~$$

$$+ e^{-t} [u(t-1)]$$

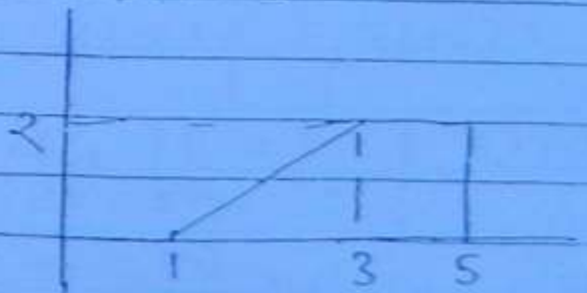
$$= \frac{1}{e}\left[\frac{1}{s^2} - \frac{e^{-s}}{s^2} \quad \frac{e^{-s}}{s}\right] + e^{-1} \cdot \frac{e^{-s}}{(s+1)}$$

$\sigma > 0$ Ans

$$= \frac{1}{e}\left[\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}\right] + \frac{e^{-(s+1)}}{(s+1)}$$

$$= \frac{1}{es^2}\left[1 - e^{-s} - s e^{-s}\right] + \frac{e^{-(s+1)}}{s+1} \quad \sigma > 0$$

Q. find L.T. of signal f(t) given below Ans



$$(t-1)[u(t-1) - u(t-3)] + 2[u(t-3) - u(t-5)]$$

$$= \frac{e^{-s}}{s^2} - \frac{e^{-3s}}{s^2} + \cancel{\frac{2}{s} e^{-3s}} + \cancel{\frac{2e^{-3s}}{s}} - \frac{2e^{-5s}}{s}$$

$\sigma > 0$ a S

Q. Find the I.L.T. of $F(s) = \frac{e^{-(s+1)}}{s^2-2s+5} = \frac{e^{-(s+1)}}{(s-2)^2+4}$

$\boxed{T22}$

$= \quad = \frac{e^{-(s+1)}}{(s-1)^2+4}$

$e^{-1} \times \frac{e^{-1}}{2} \cdot e^{t} \sin 2(t-1) \cdot u(t-1)$ Ans

~~$= \frac{1}{2} e^{(t-2)} \sin 2(t-1)\, u(t-1)$ Ans~~ $\sigma > 1$, causal system's impulse respons

$- \frac{e^{-2} \times e^{t}}{2} \sin 2(t-1) \quad u(-t+1)$ Ans

$- \frac{1}{2} e^{(t-2)} \sin 2(t-1) \quad u(-t+1) \rightarrow \sigma < 1$ (excluding $-\infty$) corresponds to stable system's impulse response

Multiplication by t property

$f(t) \longleftrightarrow F(s)$

$t^n f(t) \Leftrightarrow (-1)^n \frac{d^n}{ds^n} F(s)$ ROC remains same

$t^n u(t) \longleftrightarrow \frac{\lfloor n}{s^{n+1}} = \frac{\lceil n+1}{s^{n+1}}$

Q. Find I.L.T. of $F(s) = \frac{1}{(s-2)(s+3)^2}$ $\quad \frac{1}{\lfloor n-1} \frac{d^{n-1}}{ds^{n-1}} \frac{1}{s-2}$

$-\frac{1}{(s-2)^2}$

$= \frac{A}{s-2} + \frac{B}{(s+3)} + \frac{C}{(s+3)^2}$ $\quad B = -1/25$

$1 = A(s+3)^2 + B(s-2)(s+3) + C(s-2)$

$A = 1/25$, $\quad 1 = \frac{9}{25} + \frac{10}{25} - 6B$

$C = -1/5$

$6B = -\frac{6}{25} = -1/25$

Linearity :

Time shifting Property :→

$f(t) \longleftrightarrow F(S)$

$f(t-t_0) \longrightarrow e^{-St_0} F(S)$

$f(t+t_0) \longrightarrow e^{+St_0} F(S)$

$\delta(t) \longrightarrow$ ~~$\delta(t-t_0)$~~ 1

$\delta(t-t_0) \longrightarrow e^{-St_0}$ entire S plane ~~to~~ excluding $S \to -\infty$

$\delta(t+t_0) \longrightarrow e^{St_0}$ entire S plane excluding $S \Rightarrow \infty$

$e^{-at} u(t) \longrightarrow \frac{1}{S+a}$, $\sigma > -a$

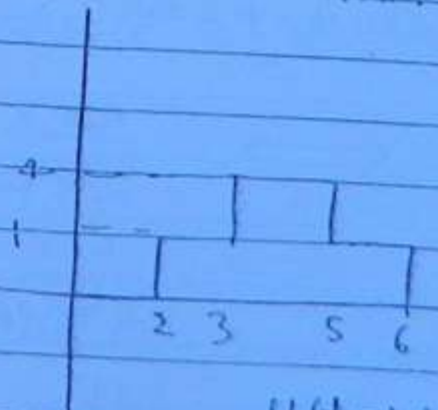
$e^{-a(t-1)} u(t-1) \longrightarrow \frac{1}{S+a} e^{-S}$, $\sigma > -a$

$\because$ shifting right will cancel the $\sigma \to -\infty$ from ROC.

but $-\infty$ is already excluded from ROC so ROC remains same

* If a signal f(t) to represent impulse response of a ~~stable~~ causal system ROC of it's laplace transform must be right sided extending till ∞, including $\sigma = \infty$.

Q) Find L.T. of f(t) shown below.



$u(t-2) + 3\,u(t-3) - 3\,u(t-5) - u(t-6)$

$= \frac{1}{S} \left[e^{-2S} + 3e^{-3S} - 3e^{-5S} - e^{-6S} \right]$ Ans

* For a given laplace transform F(S), we can defined as many inverse L.T. as there no. of ROC's.

No. of I.L.T's = No. of possible ROC.

Q How many inverse L.T exist for $F(S) = \frac{5}{(S+1)^2 (S-2)(S+3)^3}$.

4 I.L.T will exist

for a given F(S) if there are n poles then there will be (n-1) two sided I.L.T. & 2 → one is right sided of finite duration & one is left sided of finite duration so total (n+1) I.L.T. will exists.

Properties

$$f_1(t) \rightarrow F_1(S)$$

$$f_2(t) \rightarrow F_2(S)$$

$$af_1(t) + bf_2(t) \rightarrow aF_1(S) + bF_2(S)$$

$ROC \; \Phi \; ROC_1 \cap ROC$

② $e^{j\omega_0 t} f(t) \rightarrow F(S - j\omega_0)$

ROC will remain sa

③ $e^{-j\omega_0 t} f(t) \rightarrow F(S + j\omega_0)$ ROC will remain same.

$$\frac{1}{2} e^{j\omega_0 t} u(t) + \frac{1}{2} e^{-j\omega_0 t} u(t)$$

$$\cos\omega_0 t \, u(t) \longleftrightarrow \frac{S}{S^2 + \omega_0^2} \quad \sigma > 0$$

$$\sin\omega_0 t \, u(t) \longleftrightarrow \frac{\omega_0}{S^2 + \omega_0^2} \quad \sigma > 0$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(S)\, e^{St}\, dS.$$

find inverse laplace transform of $F(S) = \frac{1}{(S^2+5S+6)}$

(125)

$$= \frac{1}{(S+3)(S+2)}$$

$$= \frac{1}{S+2} - \frac{1}{S+3}$$

$$= e^{-2t} u(t) - e^{-3t} u(t)$$

Ans $\sigma > -2$

or $= -e^{-2t} u(-t) + e^{-3t} u(-t)$

$\sigma < -3$

Ans

or $-e^{-2t} u(t) - e^{-3t} u(t)$

ROC

$-3 < \sigma < -2$

Ans

Q. Find I.L.T. of the above F(S) if $S = -\frac{1}{2} + j3/2$ is in ROC. i.e L.T. is defined

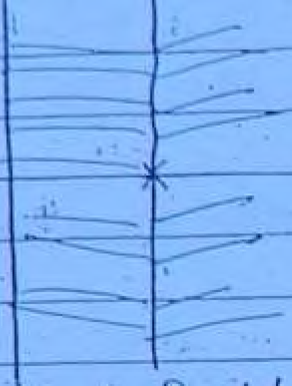
$$f(t) = e^{-2t} u(t) - e^{-3t} u(t) \quad \text{Ans}$$

$$(\sigma > -2)$$

* If $-\frac{1}{2} + j3/2$

* at $s = -1/2 + j3/2$ for L.T. to be defined this must be present in ROC.

Q. $f(t) \rightarrow F(S)$



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a signal $f(t)$ has laplace transform $F(S)$ with exactly two poles as $S = -2$ & $S = +1$ another signal $g(t)$ is defined as $g(t) = \underline{e^{+3t}} f(t)$, if $g(t)$ can represen impulse reponse of a stable system $\underline{f(t) \text{ is}}$

is stable

Newpoles 1   4 $\rightarrow e^{3t} f(t)$

if $e^{3t} f(t) \rightarrow$ left sided   Newpole -5 $-2 \rightarrow e^{-3t} f(t)$

Ans

$\longrightarrow$ ~~Right sided if~~ $e^{-3t}\ f(t)$ is stable

Q. Repeat the above question if $g(t)$ is defened as $g(t) = e^{-t} f(t)$

newpols -3 0

ⓐ $g(t)$ can't define impulse respon of a stable system

$$F(\sigma + j\omega) = F(S) = F.T.[f(t) e^{-\sigma t}]$$

$$f(t) e^{-\sigma t} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\sigma + j\omega)\, e^{+j\omega t}\, d\omega$$

$$\boxed{f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(S)\, e^{St}\, d\omega}$$

$$f(t) = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} F(S)\, e^{St}\, dS$$

$$f(t) = j \int^{j\omega} F(S)\, e^{St}\, dS$$

(*) Combination of right sided signal & left sided signal which can be ~~be~~ called as two sided signal will have finite strip in ROC bounded on the lower side by the greatest poles of all right sided signal and on the higher side by least poles all the left sided signals.

(127)

$e^{2t}u(t) + e^{5t}u(t) + e^{-2t}u(t) + e^{-5t}u(t)$

$\frac{1}{(s-2)} + \frac{1}{(s-5)} + -\frac{1}{(s+2)} + -\frac{1}{s+5}$

$\sigma > 2$ $\quad$ $\sigma > 5$ $\quad$ $\sigma < -2$ $\quad$ $\sigma < -5$

∴ does not exist because no common values of σ for which all laplace transform exist.

laplace transform of a dc signal $f(t)=1$

$$= \int_{-\infty}^{\infty} |e^{-\sigma t}|\, dt < \infty \rightarrow \text{not possible}$$

this condition will not be satisfied for any σ therefore laplace transform does not exist

Sgn(t) ⟷ not defined

$f(t) = e^{-b|t|} = e^{-bt}u(t) + e^{bt}u(t)$

$$\text{L.T.} = \frac{1}{s+b} - \frac{1}{(s-b)} = \frac{-2b}{(s^2-b^2)}$$

$\sigma > -b$ $\quad$ $\sigma < b$ $\quad$ $= \frac{2b}{(b^2-s^2)}$

ROC → $-b < \sigma < b$

$b > 0$ then it is possible.

$e^{at}[u(t) - u(t-1)] = \frac{e}{(S-a)} - \frac{e^{-as}}{(S-a)} \longrightarrow$

any pole, so all the poles of laplace transform of impulse response of such a causal & stable system must be lying on the left of s-plane.

$e^{2t} u(t) = \frac{1}{(S-2)}$ R8 —

$e^{2t} u(t) + e^{5t} u(t) \longrightarrow \frac{1}{(S-2)} + \frac{1}{(S-5)}$ ROC $\sigma > 5$

(*) Combination of any no. of right sided signal will always have defined laplace transform, which is some of all the individual Laplace transform with an ROC which is right sided bounded by greatest among all the poles.

$-e^{2t} u(-t) = \frac{1}{(S-2)}$ $\boxed{Re(S) < 2}$ ROC.

$\sigma < 2$ ROC

(*) Any combination of left sided signals will have a defined laplace transform, which is some of all individual laplace transforms with an ROC which left sided ~~by~~ bounded by least among all the poles.

(*) $f(t) = e^{-2t} u(t) + e^{-5t} u(t) + e^{2t} u(-t) + e^{5t} u(t)$

$\frac{1}{S+2} + \frac{1}{S+5} - \frac{1}{(S-2)} - \frac{1}{(S-5)}$ Ans

$Re(S) > -2$ $Re(S) > -5$ $Re(S) < 2$ $Re(S) < 5$

$S > -2,$ $S < 2$

$-2 < Re(S) < 2$

$-2 < \sigma < 2$

if it is causal $h(t) = 0$ $t < 0$

right sided signal

and laplace transform for right sided signal ~~is~~ ~~righ~~ has ROC which should be ~~as~~ right sided.

(*) If system is both stable & causal then ROC must be right sided & include $\sigma = 0$ line and it can't include any poles so for system to be causal & stable all the poles must lie on left half of s plane (∵ ROC will be right side to poles having highest σ.

(*) ROC of left sided ~~is~~ signal is also left sided

(*) if F.T. of a signal is to be defined laplace transform of corresponding must be having an ROC including $\sigma = 0$ line or $j\omega$ axis and corresponding F.T. can be obtained by replacing s in laplace transform expression by $j\omega$.

(*) A system with impulse response $h(t)$ is called as stable system if $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$ which is the same condition for the F.T. of $h(t)$ to be defined. So ~~is~~ if a system ~~is~~ with impulse response $h(t)$ is to be a stable system it must be having a laplace transform with it's ROC including $\sigma = 0$ line

(*) If a system with impulse response $h(t)$ is to be causal $h(t) = 0$ $t < 0$ i.e $h(t)$ must be generally right sided So if a system with impulse response $h(t)$ is to be causal it's laplace transform must be having an ROC which is right sided extending till $\sigma = \infty$

(*) If a system with impulse response is to be both causal and stable ROC must be right sided including [illegible]

Calculate Laplace t. $\underline{-u(-t)} = 1/s$ ROC $\sigma < 0$

$f(t) \longrightarrow F(s) \quad Re\{s\}$

$e^{at} f(t) \longrightarrow F(s+a) \quad Re\{s - a\}$

$$e^{-at} u(t) \longleftrightarrow \frac{1}{(s+a)} \qquad Re\{s+a\} > 0$$

$$Re\{\sigma + a\} > 0$$

$$\sigma > (-a)$$

$$e^{at} u(t) \longrightarrow \frac{1}{(s-a)} \qquad \sigma > a$$

$$Re\{(s-a)\} > 0$$

$$-u(-t) \longrightarrow \frac{1}{s} \qquad Re\{s\} < 0$$

$$-e^{-at} u(-t) \longrightarrow \frac{1}{(s+a)} \qquad Re\{s+a\} < 0 \text{ ROC}$$

$$\sigma < -a \quad \text{ROC}$$

$$-e^{at} u(t) \longleftrightarrow \frac{1}{(s-a)} \qquad Re\{s-a\} < 0$$

$$\sigma < a$$

(*) For a system having impulse response $h(t)$, and having laplace transform $H(s)$, for system to be stable, ROC of $H(s)$ must include $\sigma = 0$ line

∴ for stability

$$\int_{-\infty}^{\infty} |h(t)| < \infty$$

this is also condition for, F.T. to exist and from laplace transform we can get F.T. only if it's ROC includes $\sigma = 0$ line.

So if it includes $\sigma = 0$ in its ROC, then F.T. exists and system will be stable

F.T. means which can be directly find by integration transform is defined. This region of σ value is called as region of convergence [ROC].

We specify ROC for the laplace transform in terms of σ or real part of s.

③

Find Laplace T. $f(t) = \delta(t)$

$$L[f(t)] =$$

$$\int_{-\infty}^{\infty} |f(t)\, e^{-\sigma t}|\, dt < \infty$$

$$\int_{-\infty}^{\infty} |\delta(t)\, e^{-\sigma t}|\, dt < \infty$$

~~for all~~ $\int_{-\infty}^{\infty} \delta(t)\, dt = 1 < \infty$

for all values of σ Laplace transform will be defined.

$$\therefore L[\delta(t)] = 1 \qquad \text{ROC} \rightarrow \text{all value of } \sigma.$$

Laplace transform of $u(t)$.

$$\int_{-\infty}^{\infty} |f(t)\, e^{-\sigma t}| < \infty$$

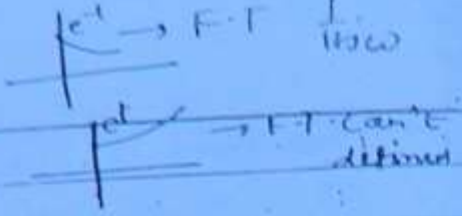
$$\int_{-\infty}^{\infty} |u(t)\, e^{-\sigma t}| < \infty$$

for $\sigma > 0$

$$\int_{0}^{\infty} |e^{-\sigma t}| < \infty$$

for $\sigma > 0 \rightarrow$ the integral converges

$L[u(t)] = \int^{\infty} e^{-st}\, dt = 1$ ROC



but F(S) must be converging.

$$|F(S)| < \infty$$

$$\left|\int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt\right| < \infty \quad \rightarrow \left(\because (S = \sigma + j\omega) \right)$$

$$\underbrace{\int_{-\infty}^{\infty} \left| f(t)\, e^{-\sigma t} \right| dt < \infty}$$

↓
condition ~~fo~~ to find ROC.

σ is real part of S
Re{S} → So ROC is defined in terms of Re{S}.

⊛ For a signal f(t) Laplace transform is defined as $= \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$

where S is a complex variable and is equal $= (\sigma + j\omega)$
So if we substitute $\sigma = 0$ in laplace transform ~~is~~ is equivalent to F.T. or laplace transform is more general form of F.T.

② We can also understand laplace transform as $F[f(t)\, e^{-\sigma t}]$ where σ is called damping ~~frequenc~~ factor.

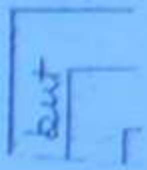
⊛ For a laplace transform to be defined, laplace transform integral must be converging i.e.

$$\int_{-\infty}^{\infty} \left| f(t)\, e^{-st} \right| dt < \infty$$

$$\int_{-\infty}^{\infty} \left| f(t)\, e^{-\sigma t} \right| dt < \infty \qquad \left(\because |e^{-j\omega t}| = \right.$$

So based on the nature of the given signal f(t) there will be a region of σ values [illegible] and laplace

Q. Repeal the previous problem for i/p PSD



$PSD_{ff} = K$

$$= \frac{1}{2\pi} K \int_{-\infty}^{\infty} \frac{1}{1+\omega^2} d\omega$$

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$$P_y \boxed{\cancel{PSD_y} = K/2}$$

$$\text{input power} = \frac{1}{2\pi} \int_{-\infty}^{\infty} K^2 d\omega$$

$$P_i = \infty$$

if f(t), g(t) are complex valued energy signal.

$$R_{fg}(\tau) = \int_{-\infty}^{\infty} f(t)\, g^*(t-\tau)\, dt$$

$$R_{ff}(\tau) = \int_{-\infty}^{\infty} f(t)\, f^*(t+\tau)\, dt$$

if f(t) & g(t) are complex valued power signal

$$R_{fg}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) g^*(t-\tau)\, dt$$

$$R_{ff}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) f^*(t-\tau)\, dt$$

$$R_{ff}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) f^*(t-\tau)\, dt$$

periodic with period T

Laplace transform :→

for a f(t) if F.T. is not defined ~~F(S)~~: by multiplying a exponential $e^{-\sigma t}$, $f(t) e^{-\sigma t}$ has a F.T.

we can choose σ value property

$$F(S) = \int_{-\infty}^{\infty} f(t) e^{-(\sigma + j\omega)t} dt = \int_{-\infty}^{\infty} f(t) e^{-St} dt$$

ROC

Q. Calculate the PSD of ~~the~~ signal $f(t) = A \sin(\omega_0 t + \phi)$

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$$R(\tau) = \frac{A^2}{2} \cos \omega_0 \tau$$

$$F[R(\tau)] = \frac{A^2}{2}\left[\pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)\right]$$

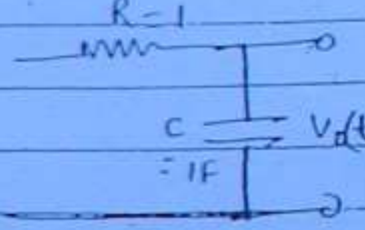
$$\text{Total power} = \frac{1}{2\pi}\int_{-\infty}^{\infty} (PSD)\, d\omega = \frac{1}{2\pi}\frac{A^2}{2}\pi \left[\int_{-\infty}^{\infty} \delta(\omega - \omega_0) + \delta(\omega + \omega_0)\, d\omega\right]$$

$$= \frac{A^2}{4}\left[1 + 1\right]$$

$$= A^2/2$$

Q. RC system given below is given an i/p signal whose power spectral density is known to be

$$PSD_f = \pi\left[\delta(\omega - 1) + \delta(\omega + 1)\right]$$



$$H(\omega) \frac{1}{j\omega + 1}$$

find Power at o/p of system

$$P_{output} = \frac{1}{2\pi}\int_{-\infty}^{\infty} PSD_f \cdot |H(\omega)|^2\, d\omega$$

$$\underline{PSD_y = PSD_f |H(\omega)|^2}$$

for LTI system

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi\left[\delta(\omega - 1) + \delta(\omega + 1)\right]\frac{1}{(\omega^2 + 1)}\, d\omega$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} \delta(\omega - 1) \cdot \frac{1}{2}\, d\omega + \int_{-\infty}^{\infty} \delta(\omega + 1)\frac{1}{2}\, d\omega\right]$$

$$P = \frac{1}{2}\left[\frac{1}{2} + \frac{1}{2}\right] = 1/2 \text{ watt}$$

Ans

$f(t) = A\cos(\omega_0 t + \phi)$ → Autocorrelation function.

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$$= \frac{1}{(2\pi/\omega_0)} \int_{-\pi/\omega_0}^{\pi/\omega_0} A^2 \cos(\omega_0 t + \phi) \cdot \cos[\omega_0 t + \phi - \omega_0 \tau]\, dt$$

$$= \frac{A^2 \omega_0}{2\pi} \left[\int_{-\pi/\omega_0}^{\pi/\omega_0} \left[\cancel{A^2} \cos^2(\omega_0 t + \phi) \cos\omega_0\tau - \cos(\omega_0 t + \phi) \sin(\omega_0 t + \phi) \cdot \sin\omega_0\tau \right] dt \right.$$

$$= \frac{A^2 \omega_0}{2\pi} \left[\int_{-\pi/\omega_0}^{\pi/\omega_0} \cos^2(\omega_0 t + \phi) \cos\omega_0\tau \, dt \right.$$

$$= \frac{[A^2 \cos\omega_0\tau]\,\omega_0}{2\pi} \left[t \, \phi + \frac{\sin 2(\omega_0 t + \phi)}{2\omega_0} \right]_{-\pi/\omega_0}^{\pi/\omega_0}$$

$$= \frac{A^2 \cos\omega_0\tau}{\cancel{4\pi}\; 2} \cdot \cancel{\omega_0} \times \frac{\cancel{2\pi}}{\cancel{\omega_0}}$$

$$R_{ff}(\tau) = \frac{A^2}{2} \cos\omega_0\tau$$

for $A\sin(\omega_0 t + \phi)$

↳ $R_{ff}(\tau) = \frac{A^2}{2} \cancel{Si} \cos\omega_0\tau$

Properties of Autocorrelation →

① $R(\tau) = R(-\tau)$ – even

② $R(0) \; \cancel{a} \geq R(\tau)$

③ $R(0)$ – Power of the signal

④ $F[R(\tau)]$ ⟶ PSD of $f(t)$

⑤ $R(\tau)$ is also periodic if $f(t)$ is periodic and period will be same as $f(t)$

~~Calculate~~

Property of Autocorrelation function →

$R(\tau)$ is even

$$R(\tau) = R(-\tau).$$

② $R(0) > R(\tau)$

③ $R(0) = E_f$

④. $\lim_{\tau \to \infty} R(\tau) = 0$

⑤ $F[R(\tau)] = ESD_f = F(\omega) F^*(\omega)$

⑥. $R_{fg}(\tau) \Longrightarrow F(\omega) \cdot G^*(\omega)$

⑦ $R_{fg}(\tau) = R_{gf}(-\tau)$

⑧ $R_{gf}(\tau) = R_{fg}(-\tau)$

(*) when $f(t)$, $g(t)$ are power signal

$$R_{fg}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t)\, g(t-\tau)\, dt$$

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t)\, f(t-\tau)\, dt$$

ff

for periodic signal

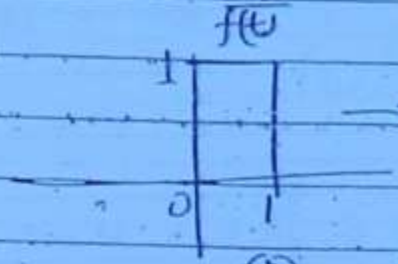
$$R_{ff}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) f(t-\tau) dt$$

T → period of periodic signal

$$R(\tau) = \int_{-\infty}^{\infty} f(t)\, f(t-\tau)\, dt$$

$$R(\tau) = \left[f(t) \otimes f(-t)\right]_{t=\tau}$$

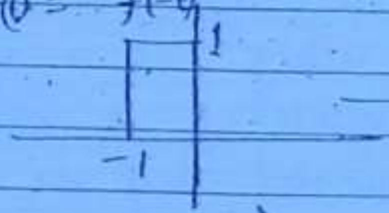
Q. Find the auto correlation of signal shown below.



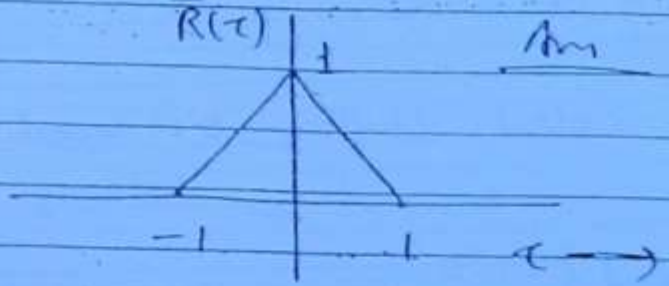
$\longrightarrow \longrightarrow 1\cdot Sa(\omega/2)\, e^{-j\omega/2}$

$\circledast$

$g(t) = f(-t)$

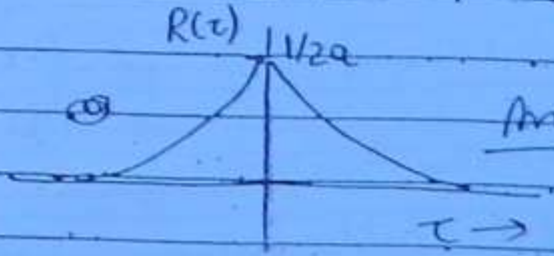


$\longrightarrow Sa(\omega/2)\, e^{j\omega/2}$



Ans ✓

Q. Find auto correlation function of $f(t) = e^{-at} u(t)$.



Ans ✓

$f(-t) = e^{at} u(t)$

$$F\left[e^{-at} \otimes e^{at} u(t)\right] = \frac{1}{(a-j\omega)} \cdot \frac{1}{(a+j\omega)}$$

$$= \frac{1}{(a^2+\omega^2)}$$

$R(t) \rightarrow \frac{e^{-a|t|}}{2a} \rightarrow R(\tau) = \frac{e^{-a|\tau|}}{2a}$ Ans

$-2a\omega_c = \ln 0.1$

$2a\omega_c = \ln 10$

$$\boxed{\omega_c = \frac{\ln 10}{2a}} \text{ rad/sec}$$

Correlation →

for two signal f(t) & g(t), if we take area under product f(t) & g(t-τ) will be larger or smaller will be acting as a ~~major~~ measure of relation b/w f(t) & g(t-τ) & correlation of the signal f(t) & g(t) denoted by the symbol

$$R_{fg}(\tau) = \int_{-\infty}^{\infty} f(t)\, g(t-\tau)\, dt$$

τ → can be called shifting or searching Parameter

$$R_{fg}(\tau) = \left[f(t) \circledast g(-t) \right]_{(t=\tau)}$$

$f(\tau)\, g(-t+\tau)$

$g(-t$

$f(\tau)\, g(\tau)$

$f(t) * g(-t)$

$$* \quad R(\tau) = \int_{-\infty}^{\infty} f(t)\, f(t-\tau)\, dt$$

The above correlation integral

The cross correlation of f(t) & g(t) defined above is mathematically similar in it's evalution to the convolution of signal f(t) & g(t).

if g(t) is happen to an even signal, functionally crosscorrelation & convolution will be same.

The above correlation if g(t) = f(t) we can defined it as autocorrelation denoted by the symbol R(τ)

Q. An ideal L.P.F. is given an i/p $f(t) = \frac{2a}{a^2+t^2}$ if the response is to have 90% of the i/p Find B.W L.P.F

$$f(t) = \frac{2a}{a^2+t^2} \qquad \frac{1}{(a+j\omega)}$$

$$\frac{1}{(a-j\omega)}$$

$$\cancel{e^{-at}u(t) \circledast e^{-at}u(t)} \longrightarrow \cancel{\frac{1}{(a^2+\omega^2)}}$$

$$e^{-at}u(t) \circledast e^{at}u(t) \longrightarrow \frac{1}{(a+j\omega)} \cdot \frac{1}{(a-j\omega)}$$

$$\longrightarrow \frac{1}{a^2+\omega^2}$$

$$e^{-a|t|} \longrightarrow \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+t^2} \xrightarrow{F.T.} 2\pi e^{-a|\omega|}$$

$$ESDf = |2\pi e^{-a|\omega|}|^2$$

$$= 4\pi^2 e^{-2a|\omega|}$$

$$\therefore \frac{4\pi^2}{2\pi}\int_{-\infty}^{\infty} e^{-2a|\omega|}\,d\omega = 2\pi\left[\left\{\frac{1}{2a}(1)\right\} + \frac{1}{2a}\right]$$

$$E_f = \frac{2\pi}{a}$$

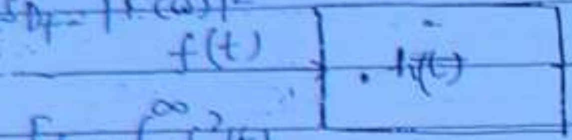
$$\therefore \frac{2\pi}{a} \times \cancel{2}\ 0.9 = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 4\pi^2 e^{-2a|\omega|}\,d\omega$$

$$\frac{0.9}{\cancel{a}} = \frac{1}{2a}\left[1 - e^{-2a\omega_c}\right] + \left(\frac{1-e^{-2a\omega_c}}{2\cancel{a}}\right)$$

$$1.8 = 2 - 2e^{-2a\omega_c} \qquad 2^{-2a\omega_c} = 0.2$$

* When the two signal are multiplied the B.W. of resultant signal is equal to some of B.W. of individual signal being multiplied.

~~$ESD_f = |F(\omega)|^2$~~



$y(t) = f(t) \otimes h(t)$

$E_f = \int_{-\infty}^{\infty} f^2(t)$

$ESD = |Y(\omega)|^2 = |F(\omega)|^2 \cdot |H(\omega)|^2$

$E_f = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$

$ESD_y = ESD_f \cdot |H(\omega)|^2$

not given assume ideal

* Q An ideal ~~LP~~ L.P.F. with cutoff frequency 1 rad/sec is given an i/p $e^{-t}u(t)$, calculate energy at response of the system

[energy at i/p = 1/2 J]

$|F(\omega)|^2 = \frac{1}{1+\omega^2} = ESD_f$

$\therefore ESD_y = ESD_f \cdot |H(\omega)|^2$

$ESD_y = \frac{1}{1+\omega^2} \cdot 1$ for $-1 \le \omega \le 1$

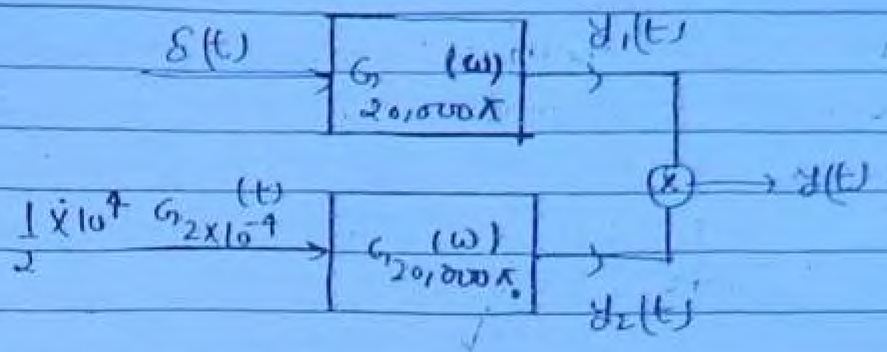
$\therefore E_y = \frac{1}{2\pi}\int_{-\infty}^{\infty} ESD_y \, d\omega$

$= \frac{1}{2\pi}\int_{-1}^{1} \frac{1}{1+\omega^2} d\omega$

$= \frac{1}{2\pi} \left(\tan^{-1}\omega\right)_{-1}^{1}$

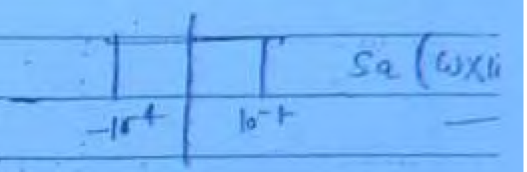
$E_y = \frac{1}{2\pi} \cdot \frac{2\pi}{4} = \frac{1}{4} J$

Ans



(1.4)

π×10⁻⁴×1×…



$Y_1(\omega) = G_{20,000\pi}(\omega)$

$Y_2(\omega) = Sa(\omega\times10^{-4}) \times G_{20,000\pi}(\omega)$

$= Sa(\omega\times10^{-4})$ from $-10,000\pi \rightarrow 10,000\pi$

B.W of $y_1(t)$ = 10,000π rad/sec

$= \frac{10,000\pi}{2\pi}$ = ~~5000~~ or 5000 ~~rad~~ Hz

B.W. $y_2(t)$ = 10,000π rad/sec

$|Y_2(\omega)| \geq \frac{1}{\sqrt{2}}$

$\left|\frac{\sin\omega\times10^{-4}}{\omega\times10^{-4}}\right| \geq 1/\sqrt{2}$

$\omega\times10^{-4}$ can't be $> \frac{1}{\emptyset}\sqrt{2}$

10^4, $\omega\times10^{-4} = \frac{\pi}{2}$

$\sin 1 \geq \boxed{\frac{1}{\sqrt{2}}}$

$10^4\times\frac{\pi}{2}$

$\frac{\sin 2}{2} \geq$ ⓟ

$\frac{1\cdot2}{\pi} \geq \frac{1}{\sqrt{2}}$

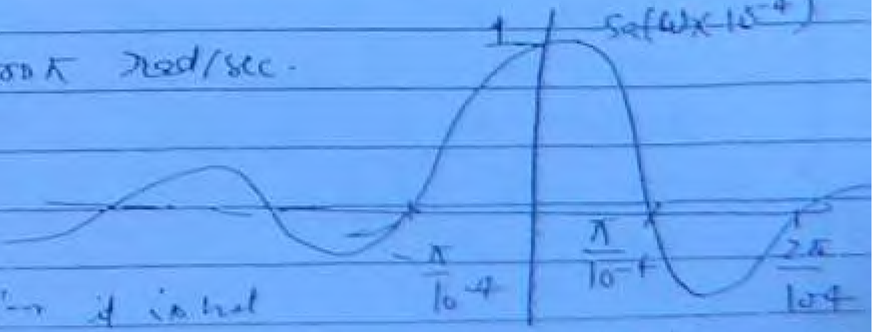
$2 \geq \boxed{\frac{\pi}{\sqrt{2}}}$

~~but this [illegible]~~

~~to ω ≥ 10,000~~

~~ω~~ >   υ <

∴ BW. Y(t) = 20,000π rad/sec.



∵ there is in question it is not given to calculate 3d.B B.W ∴ we are taking absolute B.W.

Total engy = $1/2a$

~~$\therefore \frac{1}{4a} = \int_{0}^{\omega} e^{-2at}\,dt$~~

~~$= \frac{1}{2a}\left[\;\right] = $~~

~~$F(\omega) =$~~ $\frac{1}{a+j\omega}$

$$\frac{1}{4a} = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} \frac{1}{(a^2+\omega^2)}\,d\omega$$

$$\frac{1}{2\cancel{4a}} = \frac{1}{2\pi}\left[\frac{1}{\cancel{a}}\tan^{-1}\omega/a\right]_{-\omega_c}^{\omega_c}$$

$$\frac{\pi}{2} = \tan^{-1}\omega_c/a + \tan^{-1}\frac{\omega_c}{a}$$

$$\pi/2 = 2\tan^{-1}\frac{\omega_c}{a}$$

$$\frac{\omega_c}{a} = 1$$

$\omega_c = a$ rad/sec Ans

~~ω_c~~ =

$f_c = \frac{a}{2\pi}$ Hz.

* The above ω_c value calculated can also be called as 50% energy B.W. of the signal likewise we can calculate % energy B.W. for the above signal for the specified energy required.

Q. In the following setup calculate the B.W. of signal $y_1(t)$, $y_2(t)$ & $y_3(t)$

→ means significant frequencies present in signal

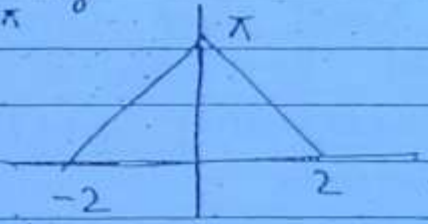
B.W. of a signal: The width of the group of frequencies for which the magnitude spectrum of a given signal is not zero is defined as BW of signal.

$f(t) = Sa(t) \xleftrightarrow{FT}$

$BW = 1$ rad/sec

$= \frac{1}{2\pi}$ Hz

$Sa^2(t) \xrightarrow{FT}$

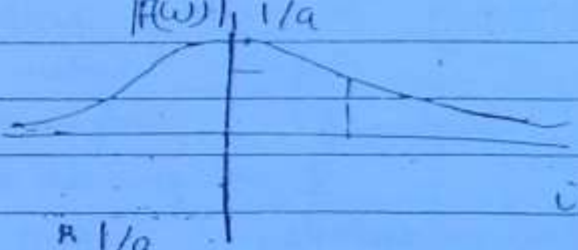


$B.W = 2$ rad/sec

$= 1/\pi$ Hz

$f(t) = e^{-at}\, u(t)$

$= \frac{1}{a+j\omega}$



B.W. ideally $= \infty$.

$|F(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$ $\frac{1/a}{\sqrt{1+\omega^2/a^2}}$

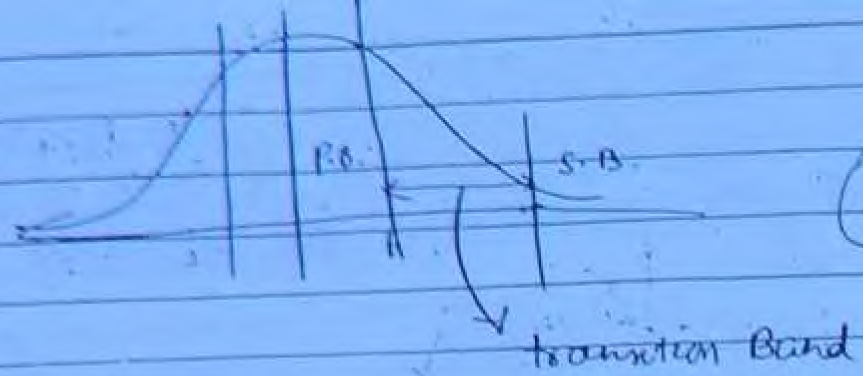
at $\omega = a \longrightarrow |F(\omega)| = \frac{1}{\sqrt{2}}\,(1/a)$

$\therefore B.W = a$ rad/sec 3 d.B B.W of signal.

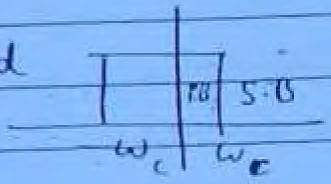
$= \frac{a}{2\pi}$ Hz. <u>Ans</u>

* 3 d.B B.W. of a signal, the width the group of frequencies for which $|F(\omega)| \geq \frac{1}{\sqrt{2}} |F(\omega)|_{max}$

Q. for a signal $f(t) = e^{-at}\, u(t)$, calculate the range of frequencies to be consider such that these frequencies considered contributes 50% total energy of signal



Δ Ideal L.P.F. has no transition band



* Ideal filters have sharp cutoff frequencies separating the pass band & stop band where as practical filters have a group of frequencies separating pass band & stop band defined transition band. Moreover the magnitude response of practically possible system can't be ~~maximum~~ absolutely flat over a range of frequency neither it can be zero for a range of frequency. ideally speaking B.W of the above system is ∞ but it is easily seen that this system is capable of making available only the frequency for which

$|H(\omega)| > \frac{1}{\sqrt{2}} |H(\omega)|max$,

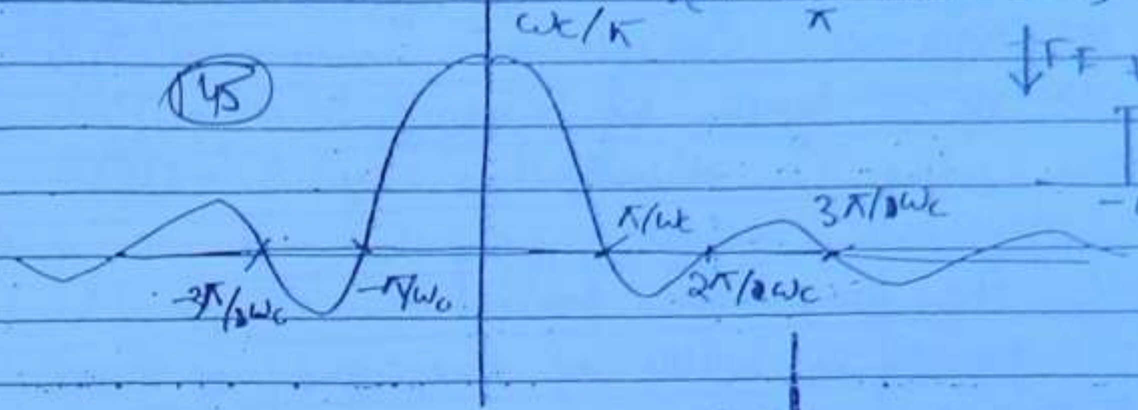
* ~~For all practical purposes~~ $|H(\omega)| > \frac{1}{\sqrt{2}} |H(\omega)|max$
i.e. Lower group of frequency of i/p at the O/P.

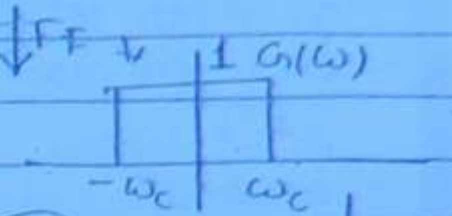
* For all practical purposes we define the width of these group of frequencies as 3dB B.W. of the system where 3dB B.W. is defined as width of those group of frequencies for which $|H(\omega)| \geqslant \frac{1}{\sqrt{2}} |H(\omega)| max$

$$\boxed{B.W = \omega = \frac{1}{2\pi RC} \text{ Hz} \quad \Big| \quad = \frac{1}{RC} \text{ Radian/Sec}}$$

$g(t) = \frac{\omega_c}{\pi} Sa[\omega_c t]$

(145)





ideal system we can't realize physically

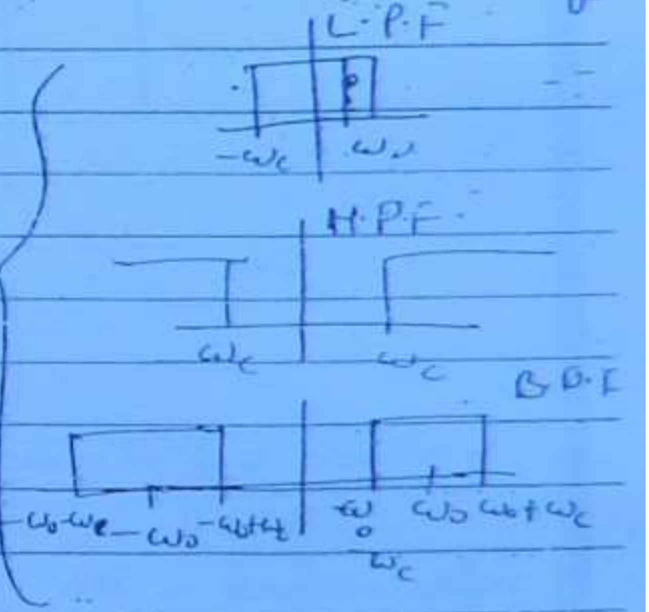
∵ $h(t) \neq 0 \quad t<0$

non causal

So we can't realize this system physically

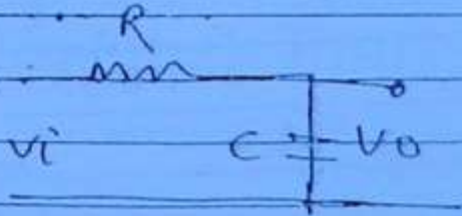
all are ideal systems

not possible

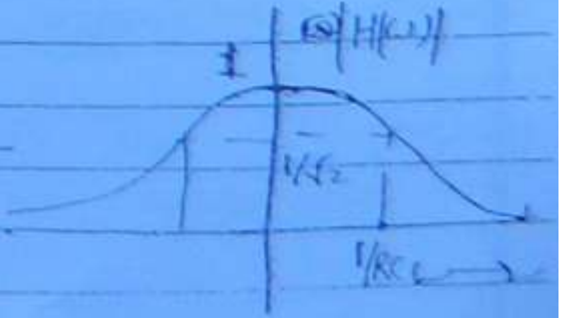


$$= \frac{\frac{1}{SC} \cdot X}{R + 1/SC}$$

$$H(\omega) = \frac{1}{RCS+1} = \frac{1}{J\omega RC+1}$$



$$|H(\omega)| = \frac{1}{\sqrt{\omega^2 R^2 C^2 + 1}}$$



$\omega << 1/RC \quad |H(\omega)| = 1$

$\omega >> 1/RC \rightarrow |H(\omega)| \rightarrow 0$

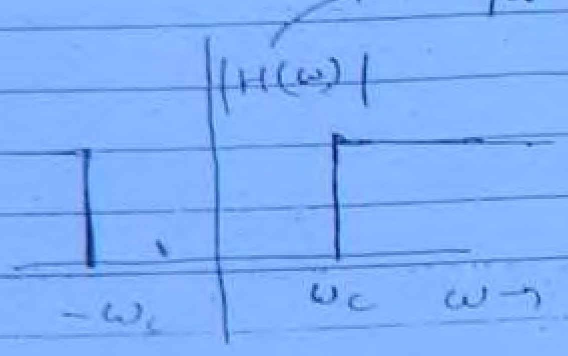
* Bandwidth of the system is defined as the width of group of frequencies which are made availble at the O/P of a system by a system. 146

A system which is distortionless system has a constant magnitude response imply that it can make available all the frequencies present in the I/P at the O/P right from 0 to ∞. So width of these group of frequencies is ∞. Hence $B\omega = \infty$

graph
for a real h(t)

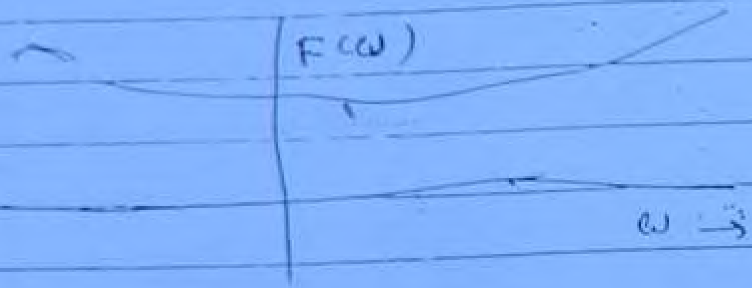
$$|H(\omega)| \qquad H(\omega)$$

is will be always even



$$|H(\omega)| = \sqrt{H_R^2 + H_I^2}$$

H_R^2: (even) fun of ω; H_I^2: (odd fun



∴ H(ω) is even conj for h(t) to be re

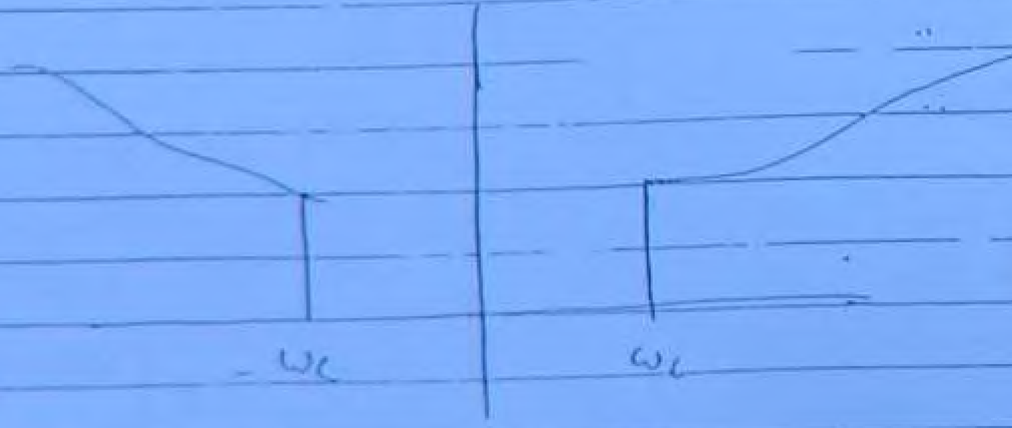
$$\therefore |H(\omega)| = \sqrt{}$$

H_R^2 → even function of ω

H_I^2 → (odd functio

= even fun

∴ |H(ω)| → evenfun of ω



B.ω = ∞

So for a real h(t) |H(ω)| will be always symmetric about y-axis.

$H(\omega) =$

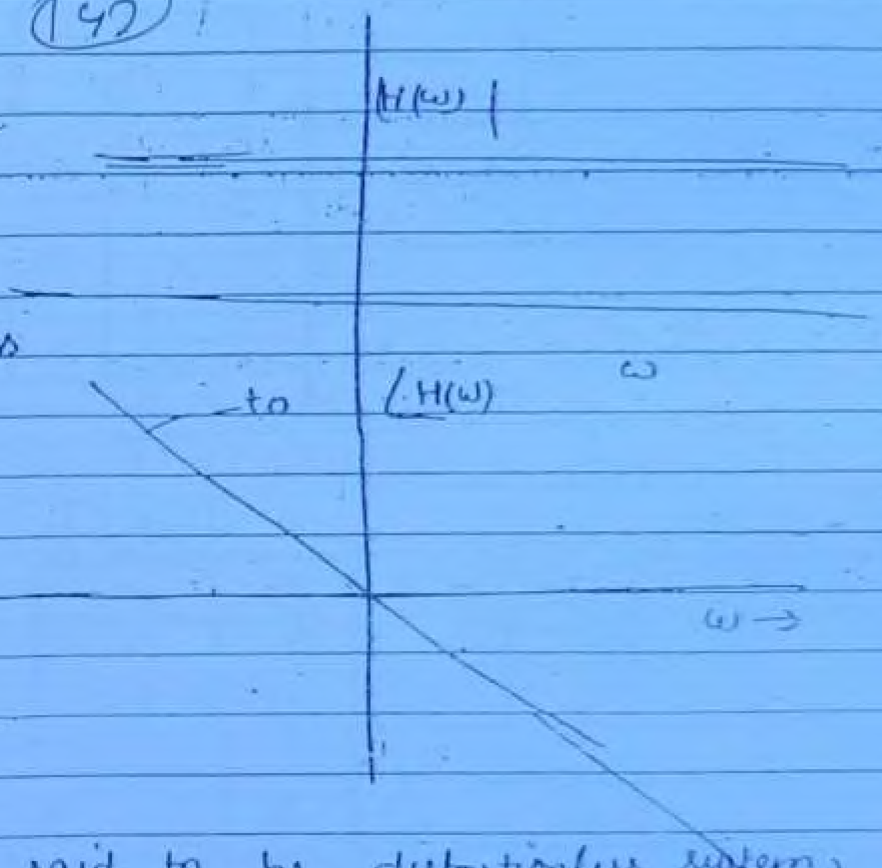
$h(t) = A\delta(t - t_0)$ → for a distortionless transmission system

$H(\omega) = A e^{-j\omega t_0} \times 2\pi$

(42)

$$\left\{ \begin{array}{l} |H(\omega)| = A \\ \angle H(\omega) = -\omega t_0 \end{array} \right.$$

condition for distortionless transmission.



* A system is said to be distortionless system, if it allows only two change in the i/p a scaler multiple a A to i/p and an uniform delay to. i.e. response to an i/p f(t) should be of the form $A f(t - t_0)$.

Transfer funct. $H(\omega) = A e^{-j\omega t_0}$ which insures constant magnitude response & linear phase response with -ve slope

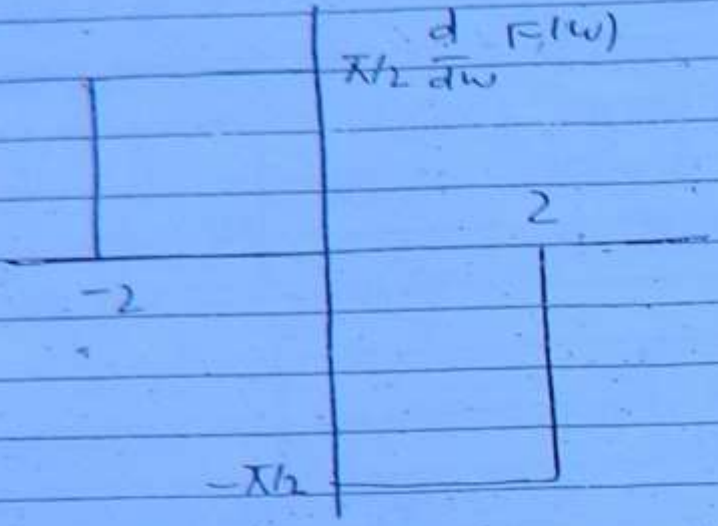
$|H(\omega)| = A$

$\angle H(\omega) = -\omega t_0$

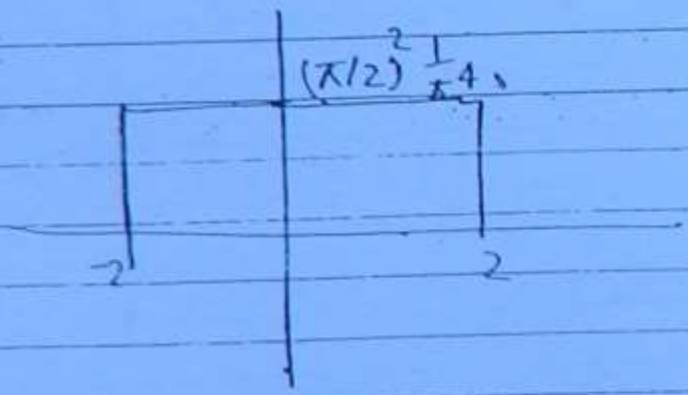
(*) Constant magnitude response insures that all frequency components present at i/p will be carried to the o/p without change in shape

(*) And linear phase response ensures sequence of information in time domain i.e all time components given uniform time delay

$F(\omega) = \frac{1}{\pi^2} j \frac{d}{d\omega} F_1(\omega)$



$|F(\omega)|^2 =$

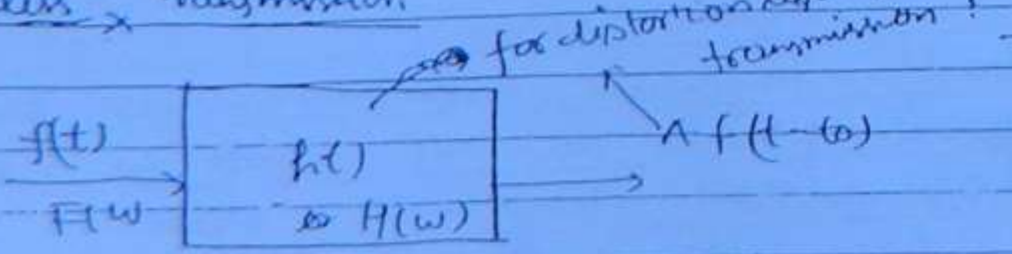


$$E = \frac{1}{2\pi} \times 4 \times \frac{1}{2\pi^3} \cdot \frac{\pi}{2}$$

$$E = \frac{\pi}{2} \frac{1}{\pi^4} \text{ Am}$$

$$= \frac{1}{2\pi^3} \text{ Am}$$

Distortionless Transmission

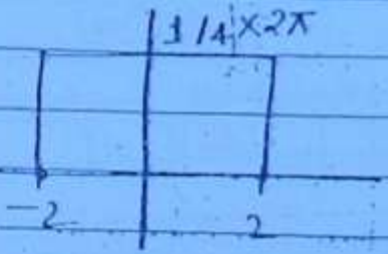


* Calculate energy of the signal $f(t) = Sa[2t]\cos 4t$

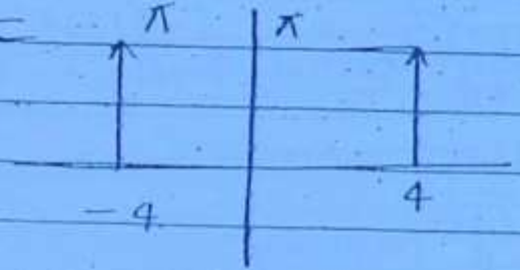
$f_1(t)$ $f_2(t)$

(149)

$f_1(\omega) =$



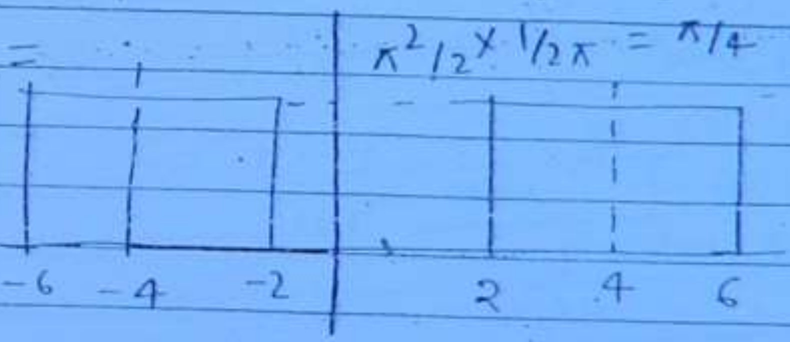
$f_2(\omega) =$



$1 = 2\pi\delta(\omega)$

$\pi\delta(\omega-4)$

$(\omega) = \frac{1}{2\pi}\, f_1(\omega) * f_2(\omega) =$

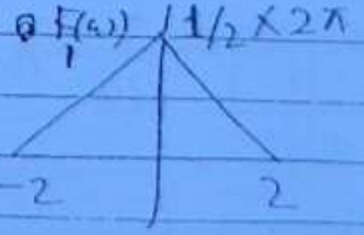


$$\int_{-\infty}^{\infty} f(t)^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\,d\omega$$

$$= \frac{1}{2\pi}\left[\frac{\pi^2}{\cancel{16}\;2} \times \overset{\pi}{\cancel{2\times 4}}\right]$$

$$E = \int_{-\infty}^{\infty} f^2(t)\,dt = \frac{\pi}{4}$$

* Calculate the energy of the signal $f(t) = t\left(\frac{\sin t}{\pi t}\right)^2$



$= \frac{t}{\pi^2}\; Sa^2(t)$

$f(t)$

$$\boxed{E = \int_{-\infty}^{\infty} f^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\,d\omega}$$

$|F(\omega)|^2$ → energy per unit frequency

$$= \int_{-\infty}^{\infty} |F(f)|^2\,df$$

$$= \int_{-\infty}^{\infty} |F(\omega)|^2\,df$$

$|F(\omega)|^2$ → energy density spectrum

or

energy spectral density

$(ESD)_f = |F(\omega)|^2$

↳ even function of frequency

$f(t) = e^{-at}\,u(t)$

$$E = 10 \cdot \frac{1}{2a}$$

$$F(\omega) = \frac{1}{a + j\omega}$$

$$|F(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$|F(\omega)|^2 = \frac{1}{a^2+\omega^2} = ESD$$

$$\therefore E = \frac{1}{2\pi}\int_{-\infty}^{\infty} o\,\frac{1}{a^2+\omega^2}\,d\omega = \cancel{\frac{1}{2}}\ \cancel{\frac{1}{2\pi a^2}}\ \frac{1}{2\pi a}\left[\tan^{-1}\omega/a\right]_{-\infty}^{\infty}$$

$$= \frac{1}{2\pi a} \cdot \pi$$

$$= \frac{1}{2a} \text{ Ans}$$

Time Integration property of F.T :

$$f(t) \longrightarrow F(\omega)$$

$$\int_{-\infty}^{t} f(\tau)d\tau \longrightarrow f(t) * u(t)$$

$$F\left[\int_{-\infty}^{t} f(\tau)d\tau\right] \longrightarrow F(\omega)\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

$$\frac{F(\omega)}{j\omega} + \pi F(\omega)\delta(\omega)$$

$$\frac{F(\omega)}{j\omega} + \pi F(0)\,\delta(\omega)$$

$$= \frac{F(\omega)}{j\omega} \quad \text{when} \int_{-\infty}^{\infty} f(t)\,dt = 0$$

$$0 = F(0)$$

(*) f(t) real valued signal

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} f^2(t)\,dt$$

$$= \int_{-\infty}^{\infty} f(t)f(t)\,dt$$

$$= \int_{-\infty}^{\infty} f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega\right]dt$$

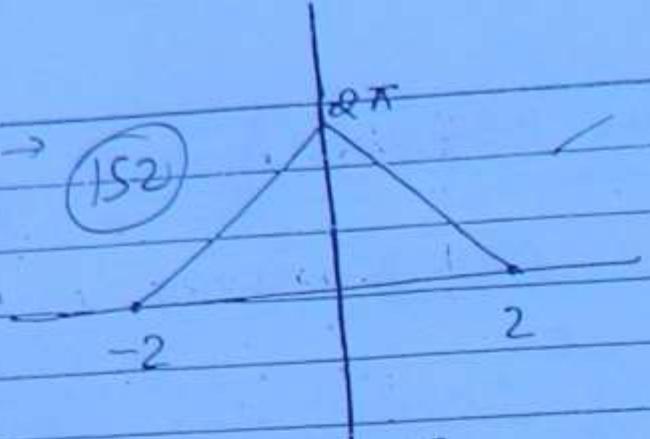
$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(t)e^{j\omega t}\,dt\right] \cancel{d\omega}\, F(\omega)d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(-\omega)F(\omega)d\omega = \frac{1}{2\pi}\int_{\infty}^{\infty} F^*(\omega)F(\omega)d\omega$$

$$-1 \int^{\infty} |F(\omega)|^2 d\omega$$

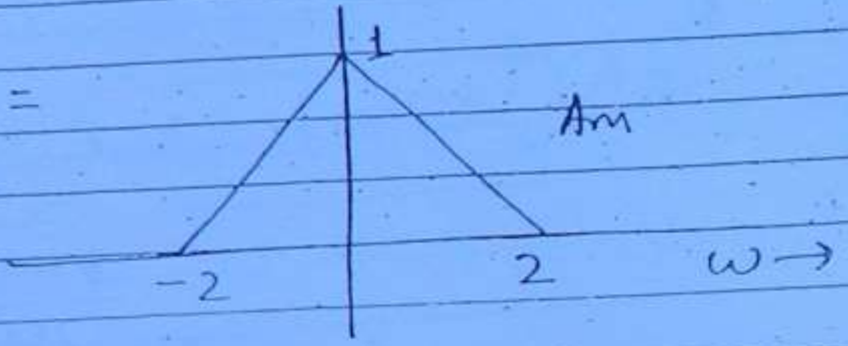
$f_1(\omega) * f_2(\omega) = \rightarrow$ ⑮②

Φ[



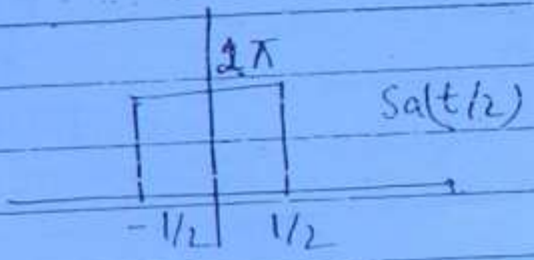
$$F[f_1(t)\cdot f_2(t)] = \frac{1}{2\pi}\left[Φ f_1(\omega) * f_2(\omega)\right]$$

=

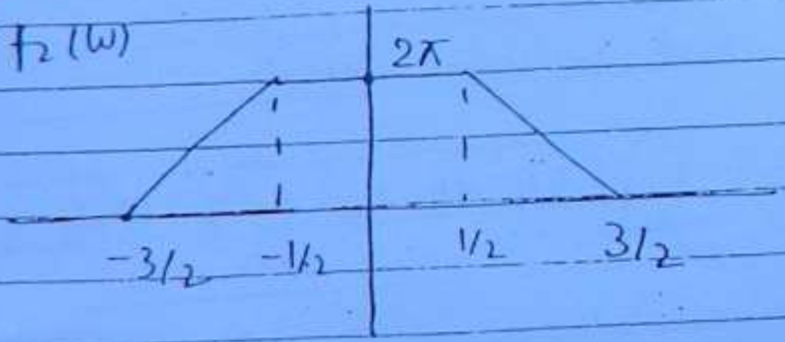


if $f_2(t) = Sa(t/2)$

$F_2(\omega) =$

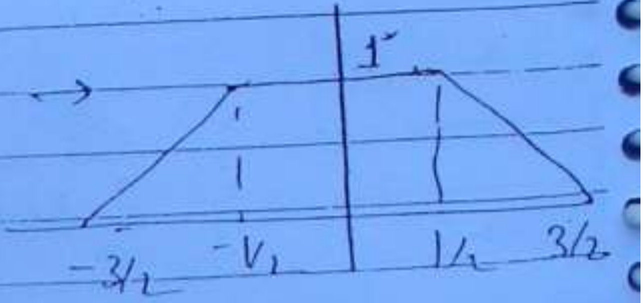


∴ $f_1(\omega) * f_2(\omega)$



(check 2π)

$$F(\omega) = \frac{1}{2\pi}\left[f_1(\omega) * f_2(\omega)\right] \rightarrow$$



$$\hat{\hat{f}}(t) = \hat{f}(t) * 1/\pi t$$

$$\hat{\hat{F}}(\omega) = -F(\omega)\, j\,\mathrm{sgn}(\omega), \quad [-j\,\mathrm{sgn}[\omega]]$$

$$= -F(\omega)\,\mathrm{sgn}^2(\omega) \qquad = -F(\omega)$$

$\hat{\hat{f}}(t) = -$ ~~$[-j\, F(\omega)\, \mathrm{sgn}(\omega)]$~~

$$\hat{\hat{f}}(t) = -f(t)$$

$$H[\sin\omega_0 t] = H[H(\cos\omega_0 t)]$$

$$= -\cos\omega_0 t$$

$$\left\{ \mathcal{H}\left[\frac{1}{\pi t}\right] = -\mathrm{sgn}^2(\omega) \right.$$

$$\left. = -1 \quad \begin{matrix} w > 0 \\ w < 0 \end{matrix} \right\}$$

$$\delta(t) \circledast \frac{1}{\pi t} = \frac{1}{\pi t}$$

$$H[\delta(t)] = \frac{1}{\pi t}$$

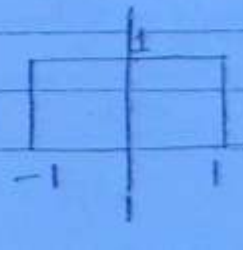
$$H[H\,\delta(t)] = -\delta(t)$$

$$H\left[\frac{1}{\pi t}\right] = -\delta(t).$$

$$f(t) = \frac{2\sin t}{\pi t^2} \cdot \sin t/2 = \frac{2}{\pi}\, \underbrace{Sa[t]}_{f_1(t)} \cdot \underbrace{Sa[t/2]}_{f_2(t)}$$

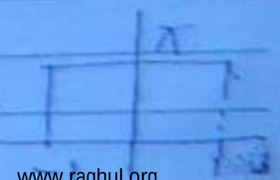
$\overset{sa(t).}{\shortparallel}$

$f_1(\omega) =$



$2\,Sa(t) \supset 2\pi\, g(\omega)$

$f_2(\omega) =$



$\frac{1}{\pi t} \longrightarrow -j\,sgn(\omega)$

$f(t) \longrightarrow F(\omega)$

$f(t) * \frac{1}{\pi t} \rightarrow F(\omega)[-j\,sgn(\omega)]$

$F(\omega)e^{-j\pi/2} = -jF(\omega) \qquad \omega > 0$

$F(\omega)e^{j\pi/2} = +jF(\omega) \qquad \omega < 0$

* Hilbert transform of a signal f(t) is denoted by a symbol $\boxed{\hat{f}(t) = f(t) * \frac{1}{\pi t}}$

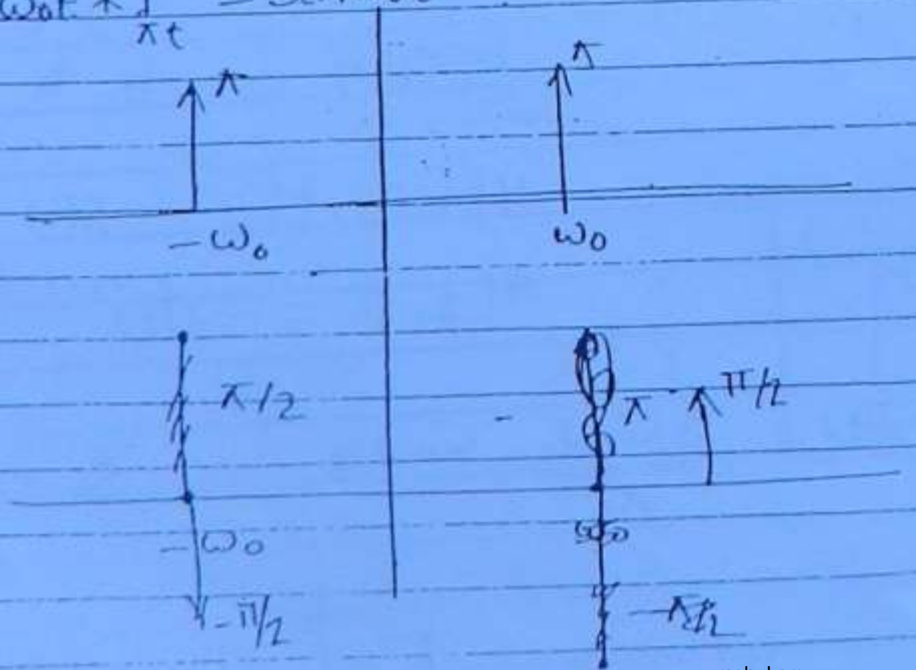
* The effect of taking Hilbert transform for a signal f(t), it shifts all frequency components of f(t) uniformly by a phase of π/2

* Hilbert transform is also called as wide band phase shifter offering a shift of π/2

Band → Group of frequencies

$f(t) = \cos\omega_0 t$

$\hat{f}(t) = \cos\omega_0 t * \frac{1}{\pi t} = \sin\omega_0 t$



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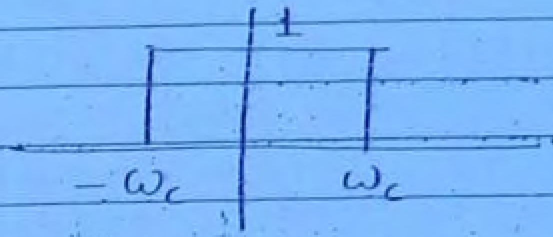
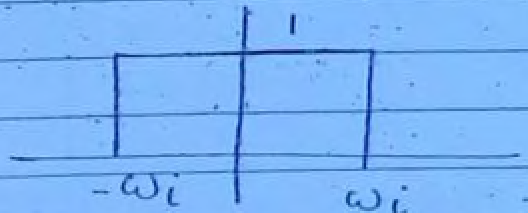
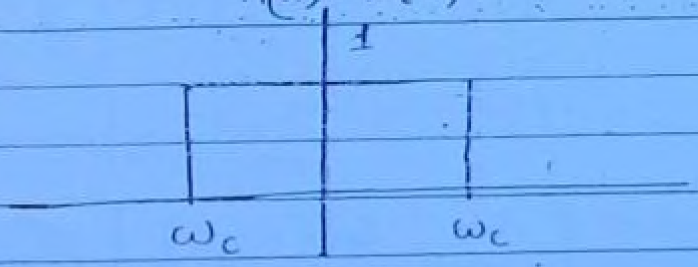
∴ y(t) $Y(\omega) = H(\omega) \cdot F(\omega)$

$= \frac{1 \times 4\pi^2}{4\pi^2} g(\omega) \cdot g'(\omega)$





$H(\omega) \cdot F(\omega)$



$\omega_i > \omega_c$

$Y(\omega) = \frac{1}{4\pi^2} 4\pi^2 \cdot g(\omega)$ if $\omega_i > \omega_c$

$= \frac{1}{4\pi^2} 4\pi^2 g'(\omega)$ if $\omega_c > \omega_i$

$y(t) = \frac{\cancel{4\pi \times 2\pi\, 2\omega_c\, Sa}[-\omega\,\omega}{\cancel{4\pi^2}}$

$= \frac{\cancel{2}}{2\pi} \cancel{2}\, \omega_c\, Sa[t\omega_c]$

$y(t) = \frac{\omega_c}{\pi} Sa[\omega_c t]$ $\omega_i > \omega_c$

$y(t) = h(t)$ $\omega_i > \omega_c$

$= f(t)$ $\omega_c > \omega_i$

$Y(\omega) = H(\omega)\,F(\omega)$ 15b

$= \frac{1}{a+j\omega} \cdot \frac{1}{b+j\omega}$ $\quad -\frac{9}{j}$

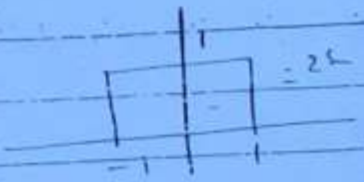
$= \frac{\cancel{(b-a)}\,1}{(b-a)(a+j\omega)} + \frac{1\,\cancel{(a-b)}}{(a-b)(b+j\omega)}$

$= \left(\frac{1}{b-a}\right)\left[\frac{1}{a+j\omega} - \frac{1}{b+j\omega}\right]$

$y(t) = \frac{1}{(b-a)}\left[e^{-at}\,u(t) - e^{-bt}\,u(t)\right]$ Ans

Q. Find the response of an LTI system

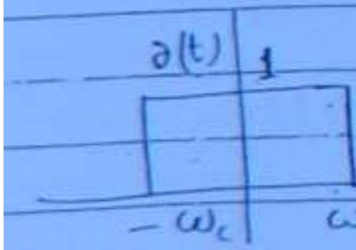
$h(t) = \frac{\sin\omega_c t}{\pi t} =$



$f(t) = \frac{\sin\omega_i t}{\pi t}$

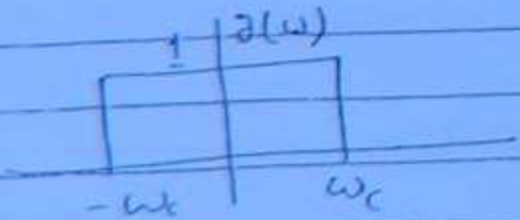
$H(\omega) = F\left[\frac{\omega_c}{\pi}\ Sa[\omega_c t]\right]$

$= F\ \frac{\omega_c}{\pi}\,[$



$g(t) \rightarrow 2\omega_c\,Sa[\omega\cdot\omega_c]$

$2\omega_c\,Sa[\omega_c t] \longleftrightarrow g(-\omega) \times 2\pi$
$g(\omega)$



$\frac{\omega_c}{\pi}\ Sa[\omega_c t] \rightarrow \frac{1}{2\pi}\,g(\omega) \times 2\pi$

$\frac{\omega_i}{\pi}\ Sa(\omega_i t) \rightarrow \frac{1}{2\pi}\,g'(\omega) \times 2\pi$

Convolution Property

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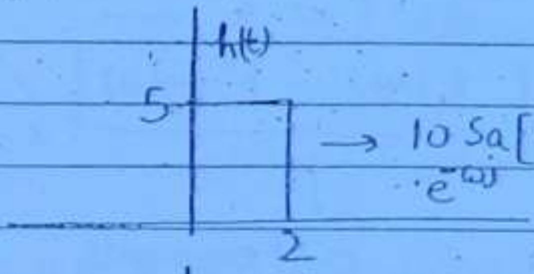
$f(t) \longrightarrow F(\omega)$

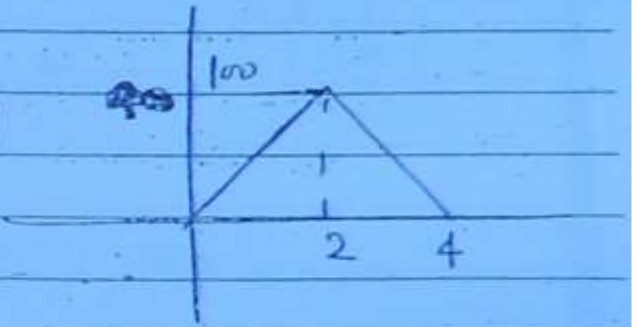
$h(t) \longrightarrow H(\omega)$

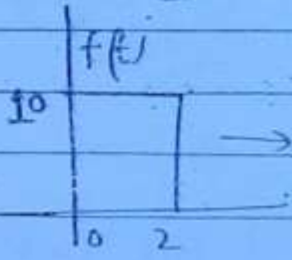
$f(t) \times h(t) \longleftrightarrow F(\omega) \cdot H(\omega)$

Q. Find response of a LTI system with an impulse response & I/P as shown below.



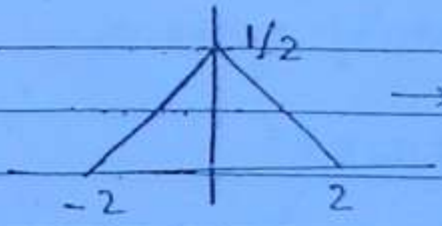
$\rightarrow 10\, Sa[\omega]\, e^{-j\omega}$





$\rightarrow 20\, Sa[\omega]\, e^{-j\omega}$

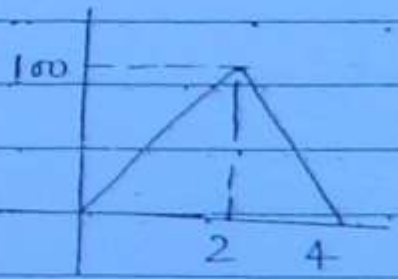
$h(\omega)\, F(\omega) \rightarrow 200\, Sa^2[\omega]\, e^{-2j\omega}$



$\rightarrow 8\, Sa^2[\omega]$

$\downarrow$

$\frac{1}{2} \times 4 \times$



Repeat the above problem for following impulse response & i/p's

$h(t) = e^{-at}\, u(t)$

$f(t) = e^{-bt}\, u(t)$

$H(\omega) = \frac{1}{a+j\omega}$

$F(\omega) = \frac{1}{b+j\omega}$

$f^*(-t) \longrightarrow F^*(\omega)$

$c = a + jb$

$C_r = a$

$j\,C_I = jb$

$$f_R(t) \quad \underset{\downarrow}{\frac{f^*(t) + f(t)}{2}} \longrightarrow \underset{\downarrow}{\frac{F^*(-\omega) + F(\omega)}{2}}$$

~~real part of f(t)~~ even conjugate of $F(\omega)$

$$j\,f_I(t) \longrightarrow \left[\underset{\downarrow}{\frac{F(\omega) - F^*(-\omega)}{2}}\right]$$

odd conjugate

$f(t) = f_R(t) + j\,f_I(t)$

$f(t) = f_{ec}(t) + f_{oc}(t)$

$$f_{ec}(t) \longrightarrow \frac{F(\omega) + F^*(\omega)}{2}$$

$\downarrow$

real part of $F(\omega)$

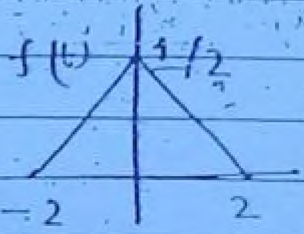
$$f_{oc}\,t \longrightarrow \frac{F(\omega) - F^*(\omega)}{2}$$

~~odd part~~ j·imaginary part of $F(\omega)$.

$$\int_{\infty}^{\infty} Sa^2(\omega)\, e^{-j\omega/2}\, d\omega$$

$$= f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{+j\omega t}\, d\omega$$

$=$



$\longrightarrow$ 2. $Sa^2[\omega]$

$\frac{1}{4}$

$$f(-1/2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{-j\omega/2}\, d\omega$$

$\frac{1}{4}t + \frac{1}{2}$

$-\frac{1}{8} + \frac{4}{8}$

$\frac{3}{8}$

$$2\pi \times 3/8 = \int_{-\infty}^{\infty} F(\omega)\, e^{-j\omega/2}\, d\omega$$

Ans $\frac{3\pi}{4}$

Symmetry Property of fourier transform

$f(t) \longrightarrow F(\omega)$

$\swarrow$ complexvalued signal

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$$

$$F^*(\omega) = \int_{-\infty}^{\infty} f^*(t)\, e^{j\omega t}\, dt$$

$$F^*(-\omega) = \int_{-\infty}^{\infty} f^*(t)\, e^{-j\omega t}\, dt$$

$$f^*(t) \longrightarrow F^*(-\omega)$$

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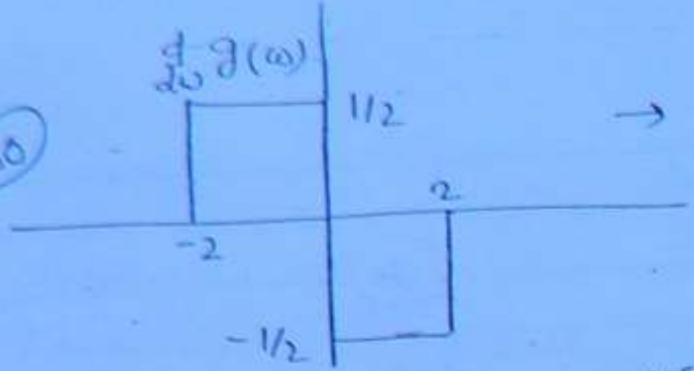


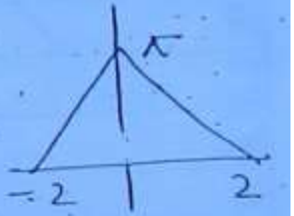
$\rightarrow \frac{1}{2} u(\omega+2) - u(\omega) + \frac{1}{2} \omega u(\omega-2)$

$$\therefore \frac{t}{\pi^2}[Sa(t)]^2 \overset{F.T.}{\longleftrightarrow} \frac{j}{2\pi}\left[u(\omega+2) - 2u(\omega) + u(\omega-2)\right]$$

Am.

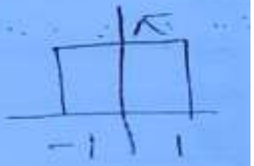
Q> $f(t) = \frac{4t}{(1+t^2)^2}$

$Sa^2(t) \longrightarrow$



$e^{-|t|} = \frac{2}{1+\omega^2}$

$Sa(t) \longleftrightarrow$



$t e^{-|t|} = -\frac{4\omega}{(1+\omega^2)^2} j$

$-\frac{4t}{(1+\cancel{\omega}t^2)^2} j \longleftrightarrow 2\pi(-\omega).\ e^{-|-\omega|}$

$\frac{4t\,j}{(1+t^2)^2} \longleftrightarrow +2\pi\omega e^{-|\omega|}$

Am

$\frac{4t}{(1+t^2)^2} \longleftrightarrow -2\pi j\omega e^{-|\omega|}$

$$\int_{-\infty}^{\infty} \frac{\sin^2 t}{\pi t^2} dt = 1$$

$$\int_{-\infty}^{\infty} \frac{1}{\pi} Sa^2(t) dt = \frac{1}{\pi} \cdot \pi = 1.$$

Duality Property :

$$f(t) \rightarrow F(\omega)$$

$$F(t) \rightarrow 2\pi f(-\omega)$$

$$\int_0^{\infty} \frac{\sin t}{t}\, dt = \pi/2$$

$$\int_{-\infty}^{\infty} \frac{\sin t}{t}\, dt = \pi$$

$\therefore \frac{\sin t}{t}$ is an even function of time

$$f(t) = \frac{1}{jt+1}$$

$$F(\omega) \Rightarrow \frac{1}{1+j\omega} \longleftrightarrow e^{-t}u(t)$$

$$\frac{1}{1+jt} \longleftrightarrow 2\pi e^{\omega} u(-\omega)$$

Q. $\frac{1}{\pi t} \xrightarrow{F.T} ?$

$$\frac{1}{j\omega} \rightarrow \frac{1}{2} sgn(t)$$

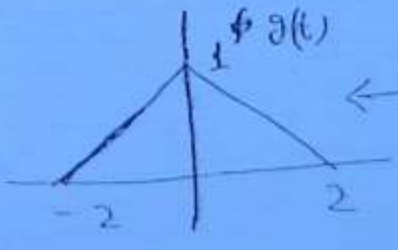
$$\frac{1}{jt} \rightarrow \frac{1}{2} 2\pi\, sgn(-\omega)$$

$$\frac{1}{\pi t} \rightarrow \frac{1}{2} 2j\, sgn(-\omega)$$

$$\frac{1}{\pi t} \rightarrow -j\, sgn(\omega)$$ Ans

Q. $f(t) = t\left(\frac{\sin t}{\pi t}\right)^2$

$$= \frac{t}{\pi^2}\left(\frac{\sin t}{t}\right)^2 = \frac{t}{\pi^2}\left[Sa(t)\right]^2$$



$$\longleftrightarrow 2\, \cancel{Sa\,t}\, [Sa(\omega)]^2$$

$$\cancel{2}[Sa(t)]^2 \longrightarrow \cancel{2}\pi\, g(-\omega) \;\; \pi g(\omega)$$

$$\therefore \cancel{2}\cdot t\, [Sa(t)]^2 \longrightarrow j\pi \frac{d}{d\omega} g(\omega) \text{ — } [illegible]$$

Area Property :→

(162)

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-J\omega t}\, dt$$

$$F(0) = \int_{-\infty}^{\infty} f(t)\, dt$$ area under curve f(t)

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{J\omega t}\, d\omega$$

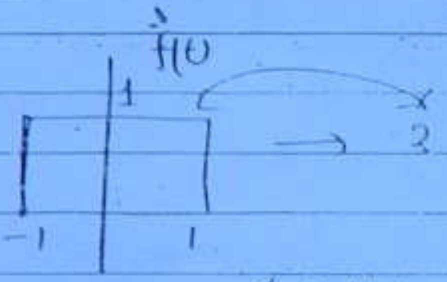
$$f(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, d\omega$$

$$\boxed{2\pi f(0) = \int_{-\infty}^{\infty} F(\omega)\, d\omega}$$

$$\int_{-\infty}^{\infty} Sa(t)\, dt$$

$$= \int_{-\infty}^{\infty} \frac{\sin t}{t}\, dt$$

$$= \pi$$



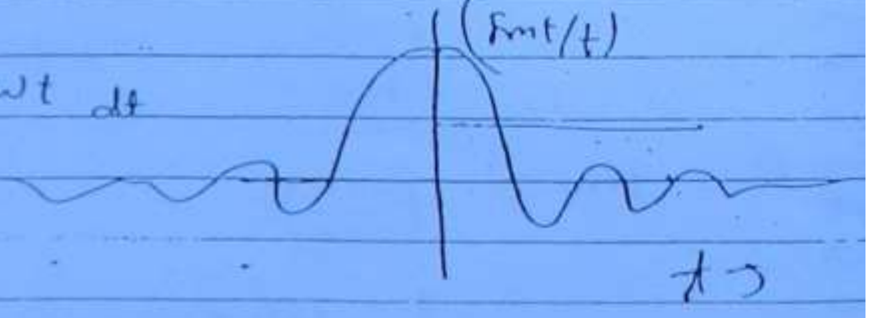
→ 2 Sa(ω·1)

$\cancel{2}\, Sa(t) \longleftrightarrow \cancel{2}\pi f(-\omega)$

∴ $\pi f(\omega)$

$$F(\omega) = \int_{-\infty}^{\infty} Sa(t)\, e^{-J\omega t}\, dt$$

$$\pi f(\omega) = \int_{-\infty}^{\infty} Sa(t)\, e^{-J\omega t}\, dt$$



$$\pi f(0) = \int_{-\infty}^{\infty} Sa(t)\, dt$$

$$\pi = \int_{-\infty}^{\infty} Sa(t)\, dt$$ Ans.

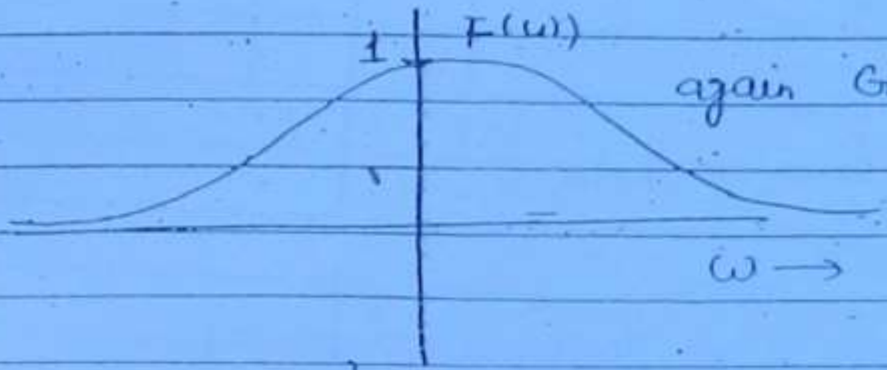
~~$F(\omega) = e^{...}$~~

$$2\pi \frac{dF(\omega)}{d\omega} + \omega F(\omega) = 0$$

$$p \; \frac{1}{F(\omega)} dF(\omega) = -\frac{\omega}{2\pi} d\omega$$

$$\log F(\omega) = -\frac{\omega^2}{4\pi}$$

$$F(\omega) = e^{-\omega^2/4\pi} = e^{-f^2\pi}$$



again Gaussian Pulse

$$e^{-\pi t^2} \longleftrightarrow e^{-\pi f^2}$$

$$\text{F.T } e^{-t^2} \longrightarrow \; ?$$

$f(t) = e^{-\pi t^2}$ ↓

~~$f\left[\frac{t\sqrt{\pi}}{\sqrt{\pi}}\right] \to 1/\sqrt{\pi}\, e^{-\omega^2/4\pi^2} \to 1/\sqrt{\pi}\, e^{-\omega^2/4\pi^2}$~~

$$f(t/\sqrt{\pi}) \longrightarrow \sqrt{\pi}\, e^{-\omega^2/4} \quad \text{Ans}$$

$$1 \longrightarrow 2\pi\,\delta(\omega)$$

$$1 \longrightarrow \delta(f)$$

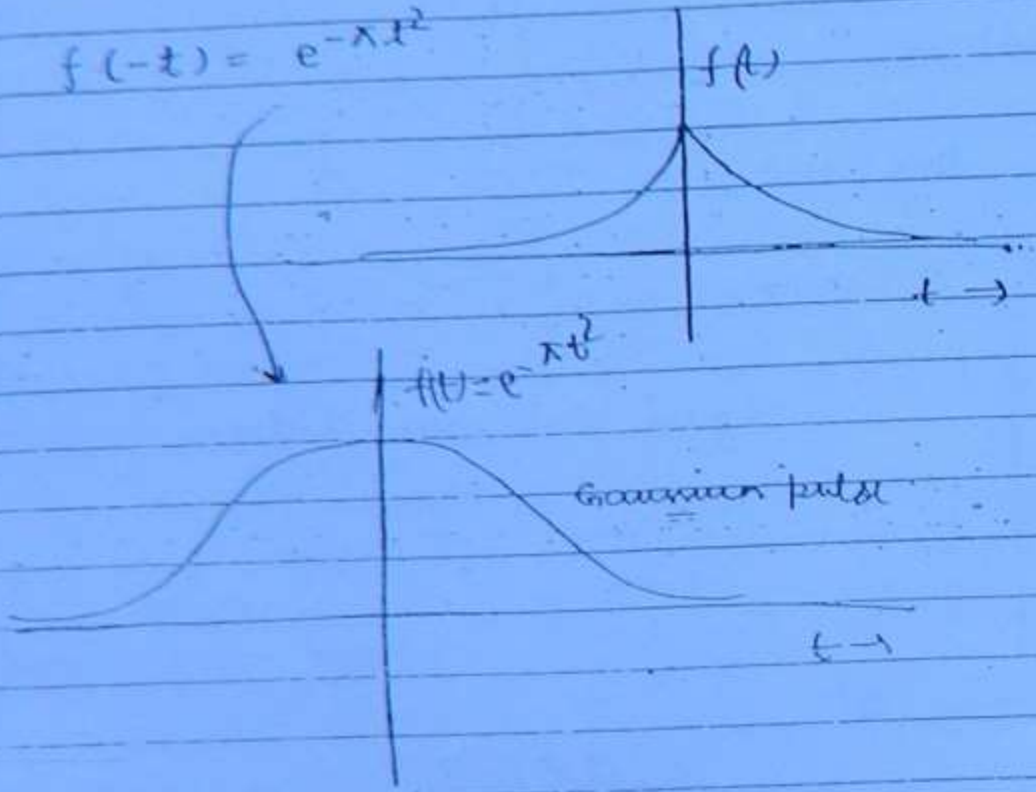
$$\text{Sgn}(t) \longrightarrow \frac{1}{j\omega} + \pi\,\delta(\omega)$$

$$\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

2 $\boxed{t^n e^{-at} u(t) \rightarrow \frac{\lfloor n}{(a+j\omega)^{n+1}}}$ ⑯④

$f(t) = e^{-\pi t^2}$

$f(-t) = e^{-\pi t^2}$



$$\frac{df}{dt} \longrightarrow j\omega F(\omega)$$

$$-2\pi t\, e^{-\pi t^2} \xrightarrow{F.T.} j\omega F(\omega)$$

$$-2\pi j \frac{dF(\omega)}{d\omega} = j\omega F(\omega)$$

$$1 \quad F(\omega)) \Big[\quad \cdot \; 0D \cdot 2\pi$$

$$2\pi \Big[\quad -2\pi \left[2\pi D \mp \omega\right] F(\omega) = 0$$

$$D = \frac{(\omega)}{2\pi}$$

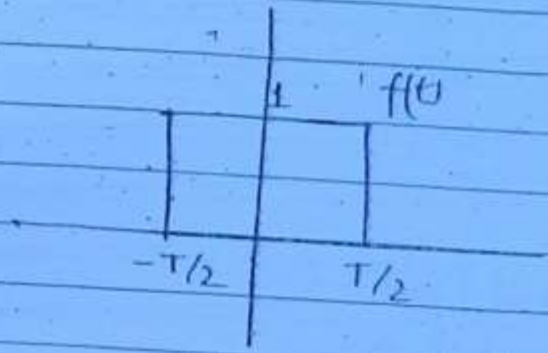
Q. $f(t) \to F(\omega)$

find F.T. of $\frac{d^2}{dt^2} f(t)$ $\frac{d^2}{dt^2}[f(t-1)]$

$f(t-1) \to e^{-j\omega} F(\omega)$ (165)

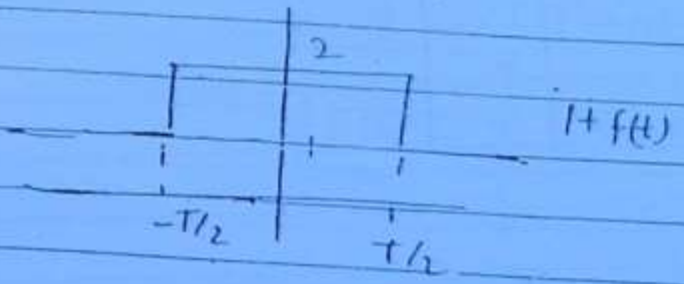
$\frac{d^2}{dt^2}[f(t-1)] \to -\omega^2 e^{-j\omega} F(\omega)$

Ans ✓



$\frac{d}{dt} f(t-1)$

d^2

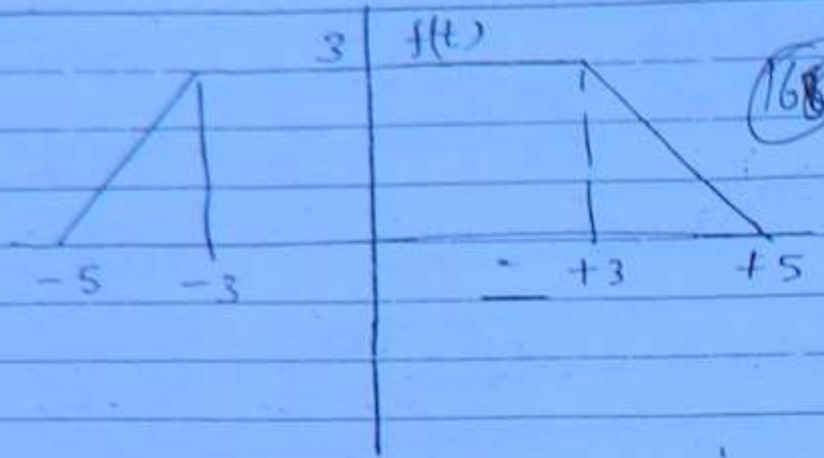


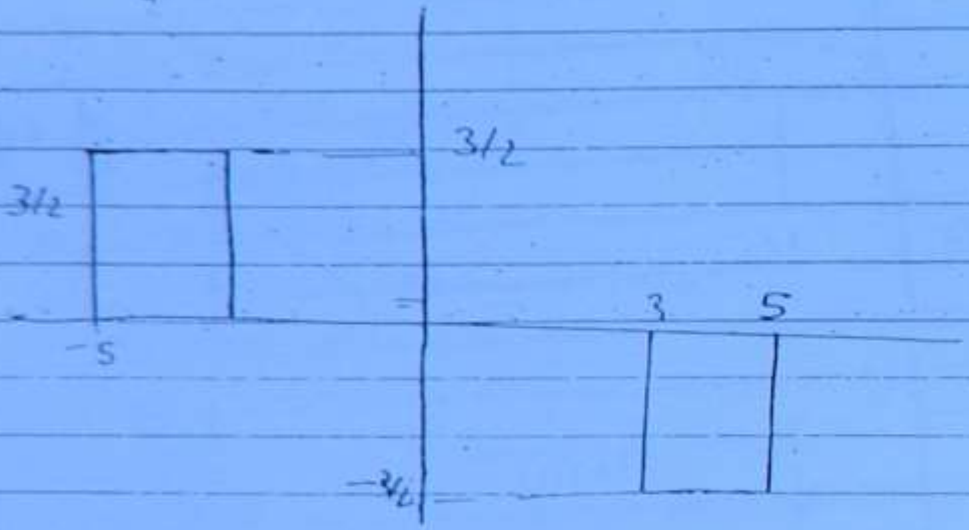
✓ f(t) & 1+f(t) are not same signal so have different F.T.
if we are going in accordance with differentiation property both have same F.T. which is not possible so there must be some condition for application of differentiation property.

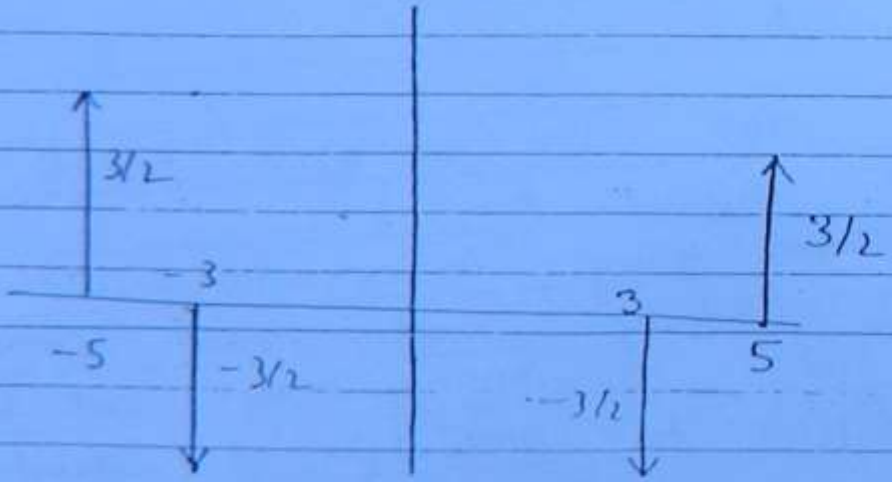
Multiplication by (t) →

$f(t) \to F(\omega)$

$t f(t) \to j \frac{dF(\omega)}{d\omega}$







$$-\omega^2 F(\omega) \longrightarrow \frac{3}{2}\left[e^{-j5\omega} + e^{+j5\omega}\right]$$

$$-\frac{3}{2}\left[e^{-j3\omega} + e^{+j3\omega}\right]$$

$$-\omega^2 F(\omega) \longrightarrow \frac{3}{2} \times 2\cos 5\omega - \frac{3}{2} 2\cos 3\omega\,]$$

$$F(\omega) \longrightarrow \frac{3}{\omega^2}\left[\cos 3\omega - \cos 5\omega\right]$$ Ans

$$F(\omega) = \frac{1}{\omega^2}\left[\frac{2}{T} - \frac{2}{T}\cos\omega T\right]$$

$$= \frac{2}{\omega^2 T}\left[1 - \cos\omega T\right]$$

$$F(\omega) = \frac{1}{\pi\omega}\left[1 - \cos\omega T\right]$$ Ans

$$= \frac{2}{\pi\omega}\sin^2\frac{\omega T}{2}$$

$$\times\frac{\omega^2 T^2}{4/2\pi\omega}$$

$$= \frac{2\times 2T}{4}\left\{Sa(\omega T/2)\right\}^2$$

$$\frac{2\pi T}{\pi}$$

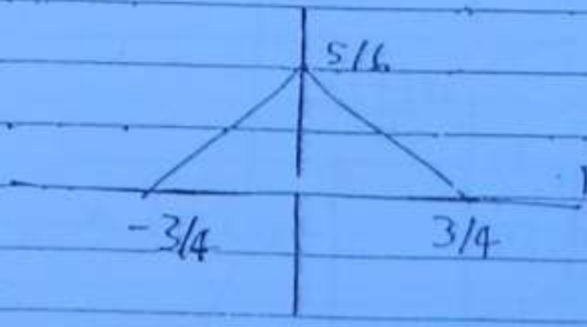
$$F(\omega) = \frac{\cancel{2}2T}{4}\left[Sa(\omega T/2)\right]^2$$

$$\frac{2}{\pi\omega}\quad\frac{\omega^2T^2}{4}$$

$$F(\omega) = \frac{2\times T}{2}\left[Sa(\omega T/2)\right]^2$$

$$\boxed{F(\omega) = \underbrace{T}_{\text{area of triangle}}\left[Sa(\omega \underbrace{T/2}_{\text{width}/4})\right]^2}$$

$$\frac{2T}{1}$$



$$\frac{1}{2}\times\frac{\cancel{3}}{2}\times 5/\cancel{6}\ 2 \qquad \frac{3}{2}$$

$$F(\omega) \rightarrow \frac{5}{8}\left\{Sa\left[\omega\cdot\frac{3}{8}\right]\right\}^2$$

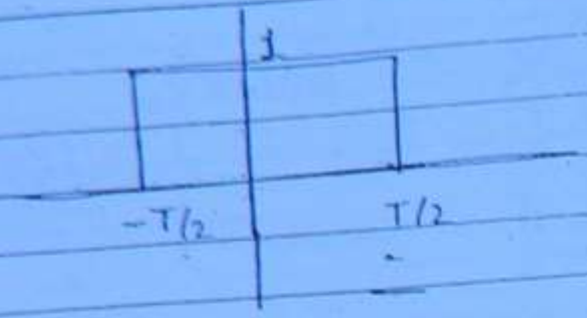
$\rightarrow G_T(t)$

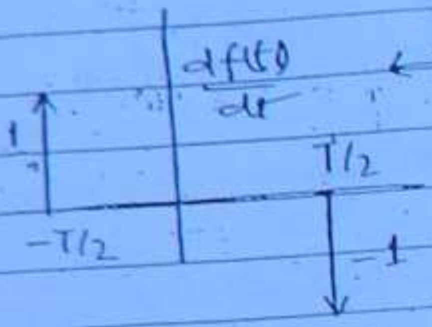
↓

only for [illegible]

T Sin ωT/2

(168)





$\longleftrightarrow \left[-e^{-j\omega T/2} + e^{j\omega T/2}\right]$

↓

$2j\sin\omega T/2$

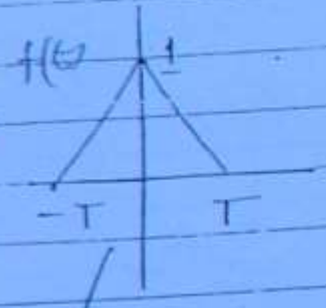
$\therefore F(\omega) \longrightarrow \frac{F\left[\frac{df(t)}{dt}\right]}{(j\omega)}$

$\longrightarrow \frac{2}{\omega}\sin\omega T/2$

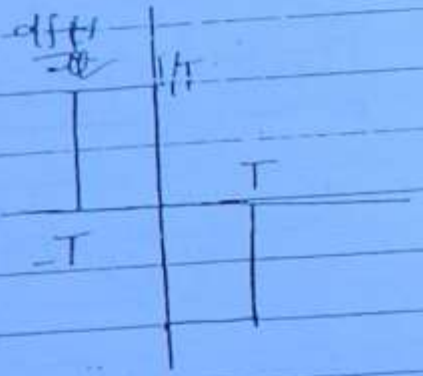
$\longrightarrow T$ ~~sin~~ $Sa(\omega T$

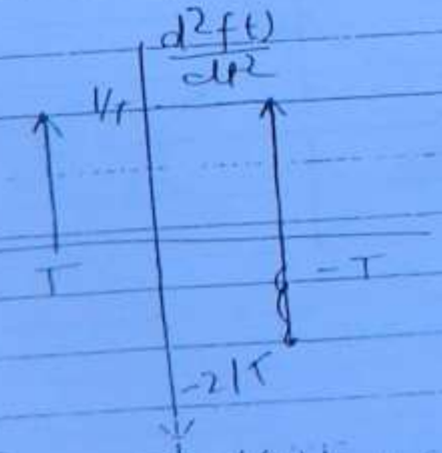
$Sa(t) \rightarrow \left(\frac{\sin t}{t}\right) =$

find F.T of



$Tri_{2T}(t)$





$(j\omega)^2 F(\omega) \longrightarrow -\frac{2}{T} + \frac{1}{T}\left(e^{j\omega T} + e^{-j\omega T}\right)$

$(j\omega)^2 F(\omega) \longrightarrow -\frac{2}{T} + \frac{1}{T} 2\cos\omega T$

Frequency shifting Property →

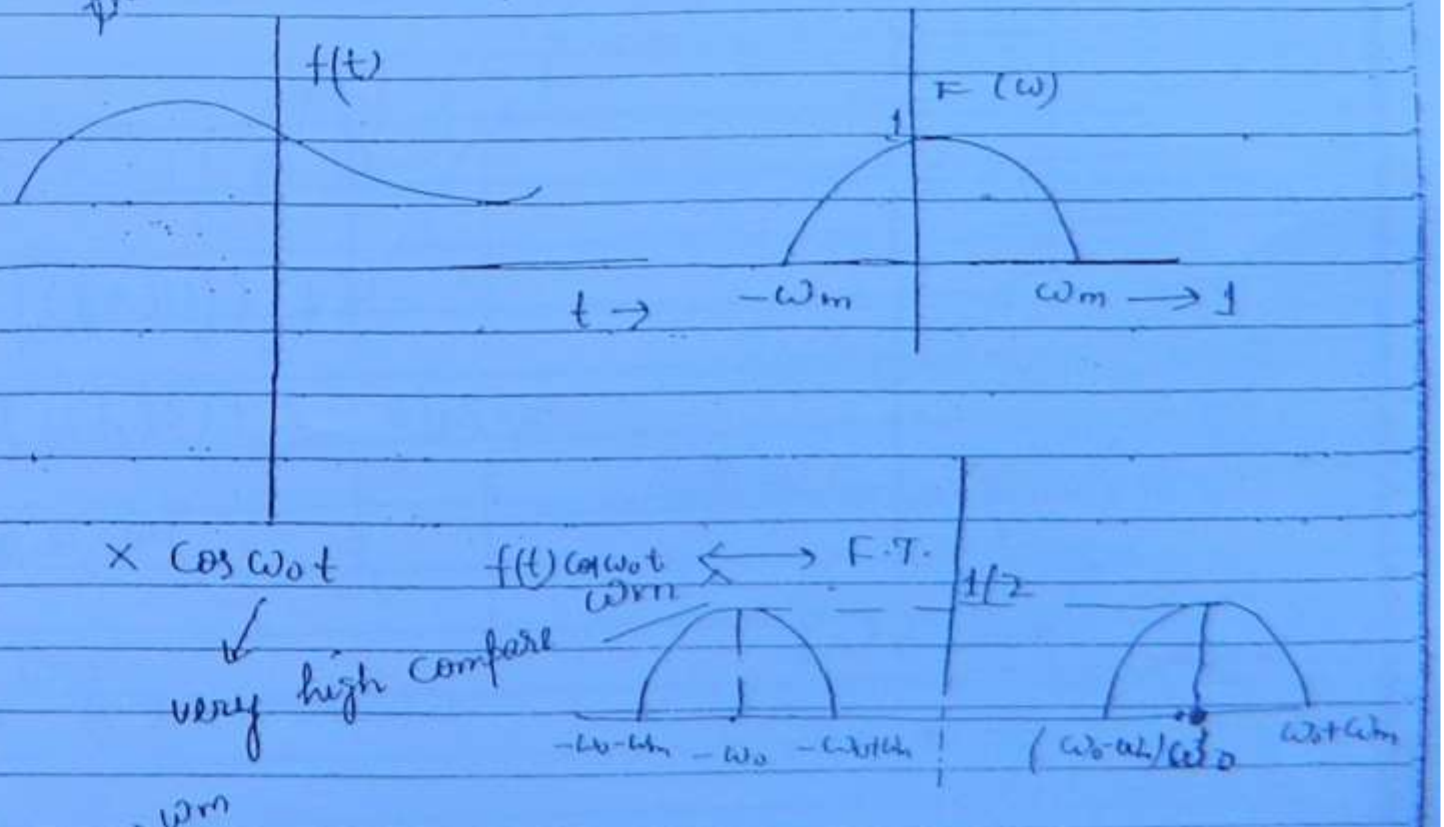
$$\boxed{e^{j\omega_0 t} f(t) \rightarrow F(\omega - \omega_0)}$$

$$e^{-j\omega_0 t} f(t) \rightarrow F(\omega + \omega_0)$$

$$\frac{1}{2} e^{j\omega_0 t} f(t) + \frac{1}{2} e^{-j\omega_0 t} f(t) \rightarrow \frac{1}{2}\left[F(\omega-\omega_0) + F(\omega+\omega_0)\right]$$

$$\boxed{f(t)\,\underline{\cos\omega_0 t} \rightarrow \frac{1}{2}\left[\underline{F(\omega-\omega_0)} + \underline{F(\omega+\omega_0)}\right]}$$

modulation theorem



$\omega_0 >> \omega_m$

Time scaling

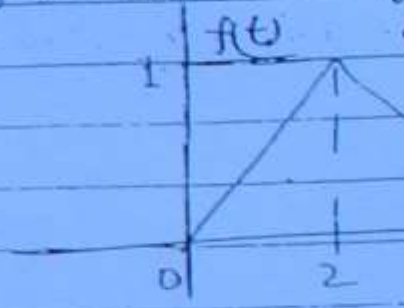
$$f(t) \longrightarrow F(\omega)$$

$$f(at) \longrightarrow \frac{1}{|a|} F(\omega/a)$$

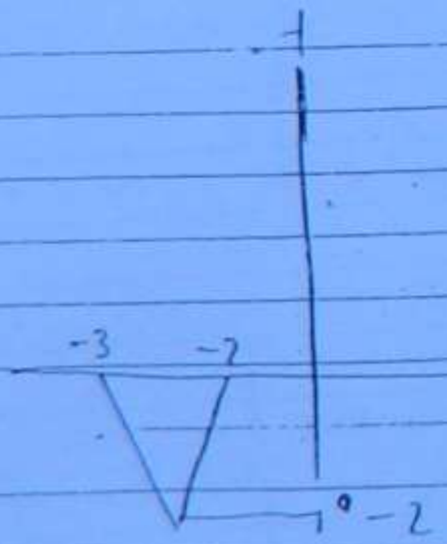
* Expansion of a signal in time domain leads to compression in frequency domain by the same proportion and vice versa.

* Always maintaining the product of time width & frequency width as a constant

Q. F.T. of f(t) shown below is known to F(ω) then find the F.T. of g(t).



$\longleftrightarrow F(\omega)$



$$f(\cancel{[illegible]}\,4t)$$

$$= \cancel{f3(}\, f[4(t+3)]$$

$$g(t) = -2 f(4t+12)$$

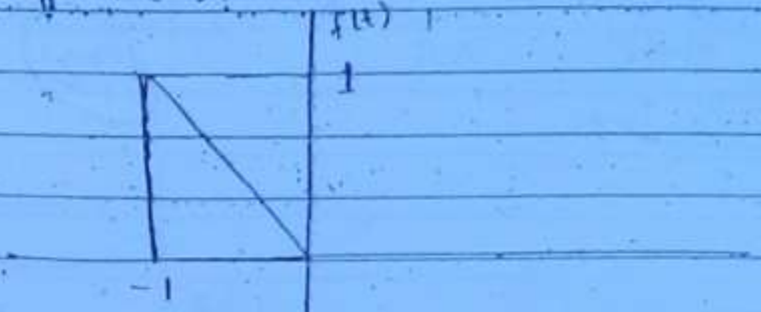
$$= \frac{-2}{4} e^{j\omega \times 12/4} F(\frac{\omega}{4}$$

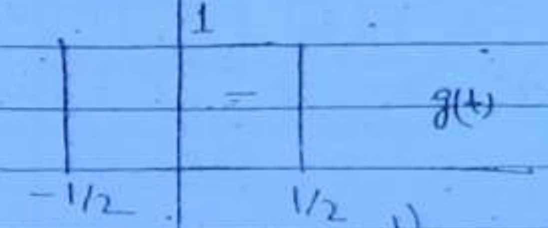
$$G(\omega) = -\frac{1}{2} e^{3j\omega} F(\omega/4$$

$$F(\omega) = \left\{\frac{1}{J\omega} + \pi\delta(\omega)\right\} \left[1 + 3e^{-J\omega\times 2} - 3e^{-J\omega\times 5} - e^{-J\omega\times 7}\right]$$

27th oct 10 ⑰

Q. If the fourier transform of the signal f(t) is known to F(ω) find F.T. of signal g(t) in terms of F(ω).

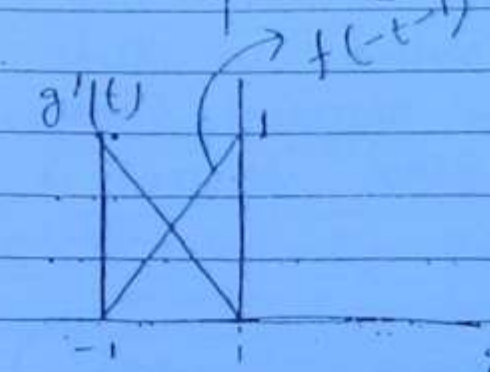




f(-t-1)

f[-(t+1)]

F(-ω)



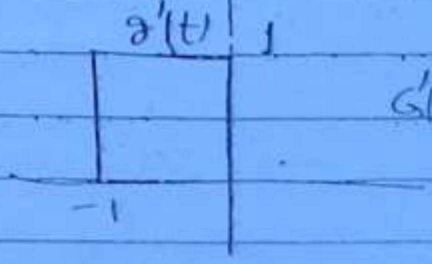
$$g'(t) = f(t) + f(-t-1)$$

$$G'(\omega)\cancel{g'(t)} = F(\omega) + e^{+J\omega_0} F(-\omega)$$

$$G(\omega) = e^{-J\omega_0 1/2} \cdot F(\omega) + e^{+J\omega_0 1/2} F(-w$$

=

Ans



* Even if F.T. is defined indirectly, for signal like sgn(t) & 1 which is not satisfying the above condition ($\int_{-\infty}^{\infty} |f(t)|dt < \infty$), the F.T. so derived will not be well defined F.T. i.e. this F.T. will have an undefined value atleast for one value of ω.

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Properties of F.T. :→

Time shifting

$$f(t) \longrightarrow F(\omega) = |F(\omega)| \angle F(\omega)$$

$$f(t-t_0) \longrightarrow e^{-j\omega t_0} F(\omega) = e^{-j\omega t_0} |F(\omega)| e^{j \angle}$$

$$|F(\omega)| \quad [\angle F(\omega) - \omega t_0]$$

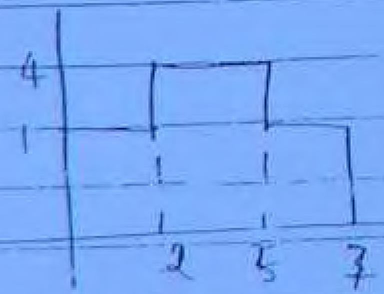
⊛ When a signal is shifted in time domain magnitude part of spectrum remains same, only phase spectrum is affected i.e. phase spectrum is changed by $-\omega t_0$

$$\delta(t) \longrightarrow 1$$

$$\delta(t-t_0) \longrightarrow e^{-j\omega t_0}$$

$$\delta(t+t_0) \rightarrow e^{j\omega t_0}$$

$$f(t-t_0) + f(t+t_0) \longrightarrow 2\cos \omega t_0 \, F(\omega)$$



$u(t) + u(t-1) - u(t$

$- u(t-3)$

$f(t) = u(t) + 3u(t-2) - 3u($

$- u(t-7)$

$$F(\omega) = \frac{1}{j\omega}\left[e^{j\omega T/2} - e^{-j\omega T/2} \right]$$

(173)

$$= \frac{1}{j\omega}\, 2j \sin \omega T/2$$

$$\sin\theta = \frac{\sin \pi\theta}{\pi\theta}$$

$$F(\omega) = \frac{2 \sin \omega T/2}{\omega} - \overset{\downarrow}{\textcircled{T}}\, Sa(\omega \overset{\downarrow}{\textcircled{T/2}})$$

T.

T/2

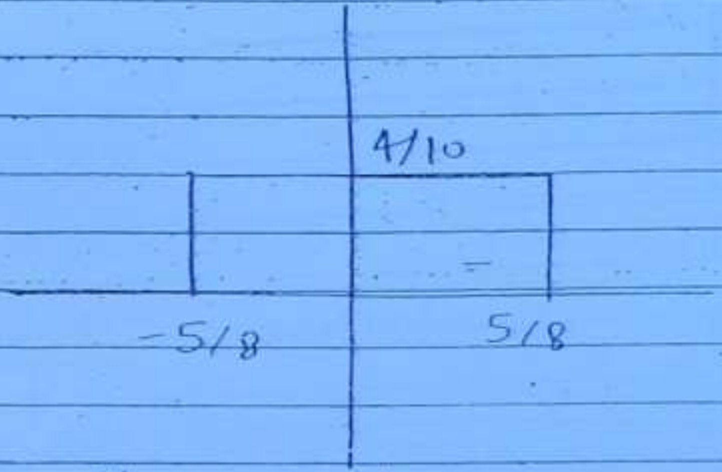
$$F(\omega) = T \cdot \operatorname{Sinc} \frac{\omega T}{2\pi}$$

area of pulse

half of width

$$\frac{T}{(\pi\, \omega T/2\pi)}$$

$\sin \pi\theta$



$$\frac{4}{10} \times \frac{2 \times 5}{8_2} \times$$

$$\frac{1}{2} Sa\left(\frac{5\omega}{8}\right)$$

$e^{-at} u(t)$, $e^{at} u(t)$, $\delta(t)$, $rect(t) \xrightarrow{F.T}$ is completely defined F.T.

$Sgn(t)$, 1, $e^{j\omega_0 t}$, $\cos\omega_0 t$, $\sin\omega_0 t$, $u(t) \rightarrow$ F.T. is not completely defined for all values of ω.

Existence of Fourier Transform →

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$|F(\omega)| < \infty$$

$$\left| \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \right| < \infty$$

~~$$\left| \int_{-\infty}^{\infty} f(t)\, dt \right| < \infty$$~~

Imp: if this condition is satisfied then only we can use the integral formula for calculation of F.T.

$$\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$$ → for well defined F.T. f(t) must satisfy this condition.

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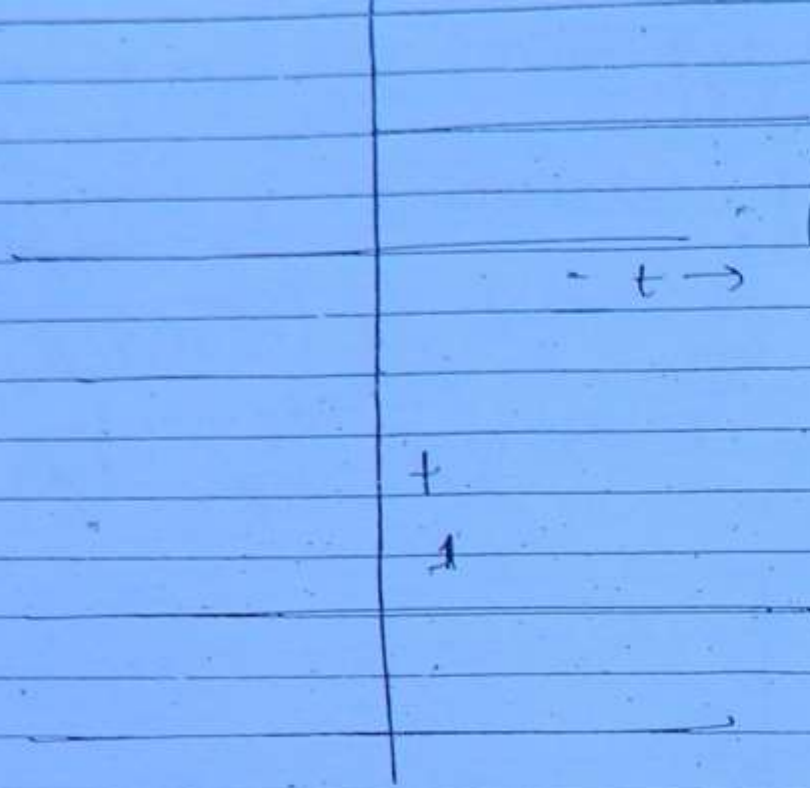

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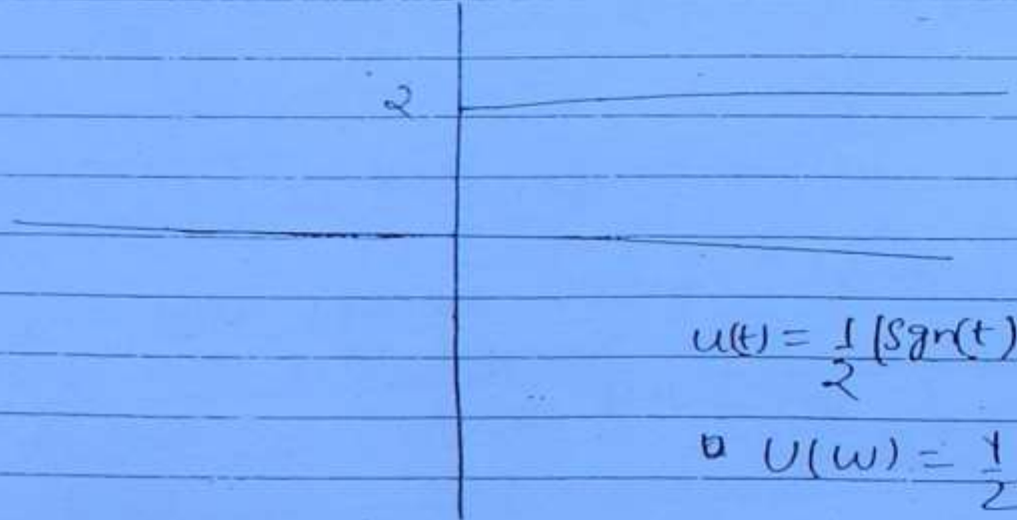
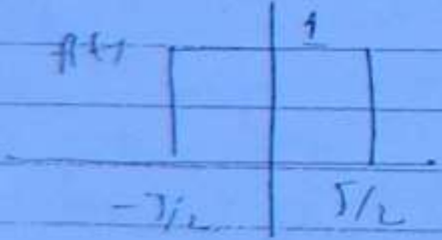


$u(t) = \frac{1}{2}[sgn(t) + 1]$

$U(\omega) = \frac{1}{2}\left[\frac{1 \;\; 2}{2j\omega} + 2\pi\delta(\omega)\right]$

$U(\omega) = \frac{1}{j\omega} + \pi\delta(\omega$



$= \int_{-T/2}^{T/2} 1 \cdot e^{-j\omega_0 t}\, dt$

$F(\omega) = \left[\frac{e^{-j\omega}}{-j\omega}\right]_{T/2}^{T/2}$

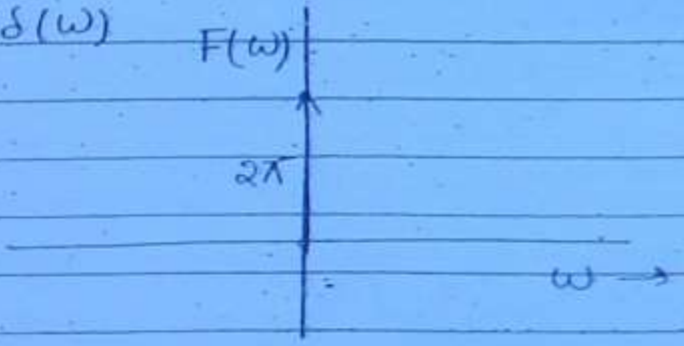
(175) $\cos\omega_0 t \longrightarrow \pi\delta(\omega+\omega_0) + \pi\delta(\omega-\omega_0)$

$\frac{1}{2j} e^{j\omega_0 t} + \frac{1}{2j} e^{-j\omega_0 t} \longrightarrow \frac{1}{2j} \; \frac{\pi}{j} \delta(\omega-\omega_0)$

$- \frac{\pi}{j} \delta(\omega+\omega_0)$

$\sin\omega_0 t \longrightarrow \frac{1}{j}\left[\pi\delta(\omega-\omega_0) - \pi\delta(\omega+\omega_0)\right.$

$1 \longrightarrow 2\pi\delta(\omega)$

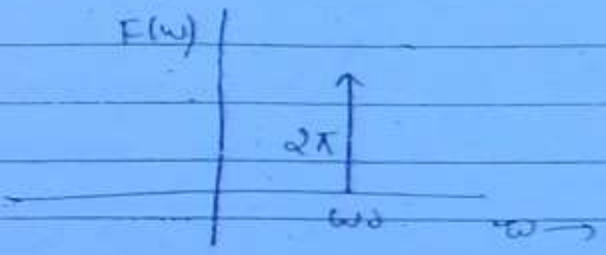


$e^{j\omega_0 t} \longrightarrow 2\pi\delta(\omega-\omega_0)$

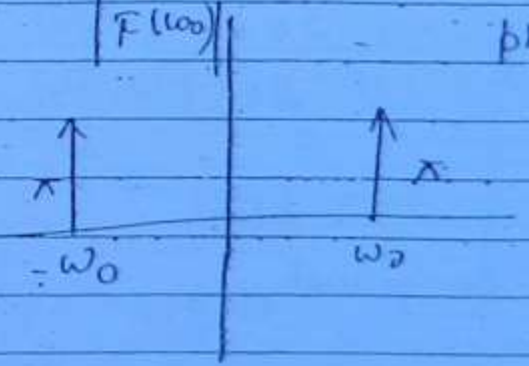
↓

having a single angular frequency

ω_0

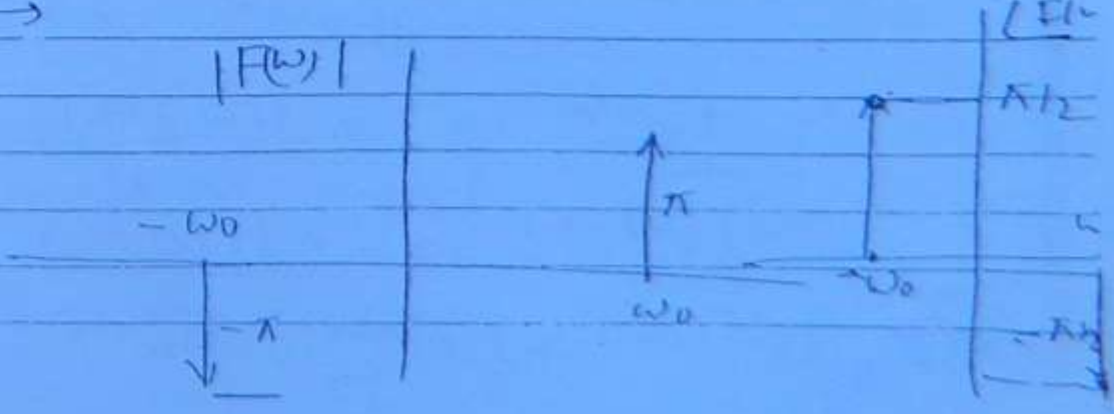


$\cos\omega_0 t \rightarrow$



phase spectrum not exist →
~~because its~~
∴ fourier transfor is real.

$\sin\omega_0 t \rightarrow$



$$\delta(t) \longrightarrow 1$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$F(\omega) = \delta(\omega)$$

$$f(t) = 1/2\pi$$

$$\frac{1}{2\pi} \longrightarrow \delta(\omega)$$

$$1 \xrightarrow{F.T.} 2\pi\delta(\omega)$$

$$F(\omega) \longrightarrow \delta(\omega - \omega_0)$$

$$f(t) = \frac{1}{2\pi} e^{j\omega_0 t}$$

$$\frac{e^{j\omega_0 t}}{2\pi} \longrightarrow \delta(\omega - \omega_0)$$

$$e^{j\omega_0 t} \longrightarrow 2\pi\delta(\omega - \omega_0)$$

$$1 \cdot e^{j\omega_0 t} \longrightarrow 2\pi\delta(\omega - \omega_0)$$

$$f(t) \longrightarrow F(\omega)$$

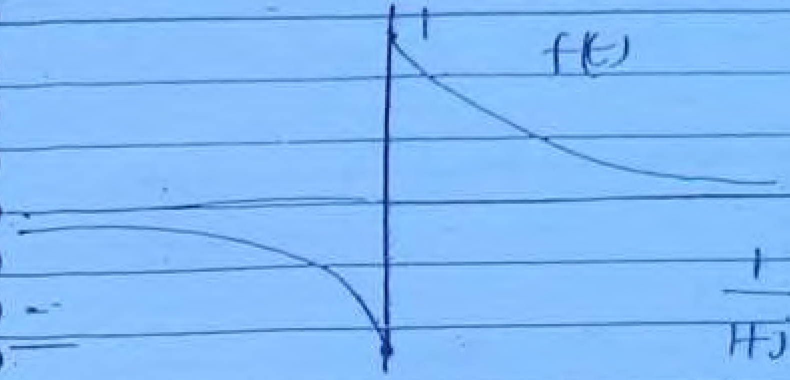
$$\cancel{f(t)}t \; e^{j\omega_0 t} \, f(t) \longrightarrow F(\omega - \omega_0)$$

$$f(t)\, e^{-j\omega_0 t} \longrightarrow F(\omega + \omega_0)$$

$$\frac{1}{2} e^{j\omega_0 t} \longrightarrow \pi\delta(\omega - \omega_0)$$

$$\frac{1}{2} e^{-j\omega_0 t} \longrightarrow \pi\delta(\omega + \omega_0)$$

$$1/2\; e^{j\omega_0 t} + \frac{1}{2} e^{j\omega_0 t} \longrightarrow \pi\delta(\omega - \omega_0) + \pi\delta($$



$$\frac{1}{1+j\omega} - \frac{1}{1-j\omega}$$

$$F(\omega) = \frac{-2j\omega}{(1+\omega^2)}$$ imaginary & odd in nature

f(t) is real & odd

↳ F(ω) is ~~also~~ imaginary & odd.

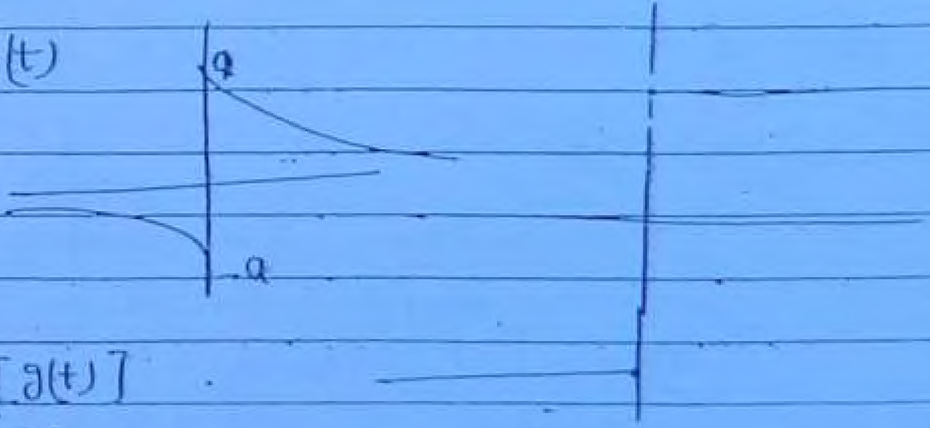
(*) Fourier transforms of neither even nor odd signals is generally ~~is~~ complex in nature this complex fourier transform is conjugate symmetric.

(*) When f(t) is an even signal Fourier transform is purely real is even function of ω.

(*) When f(t) is odd fourier transform is purely imaginary and this imaginary part is odd.

$\lim g(t) = \mathrm{sgn}(t)$

$a \to 0$

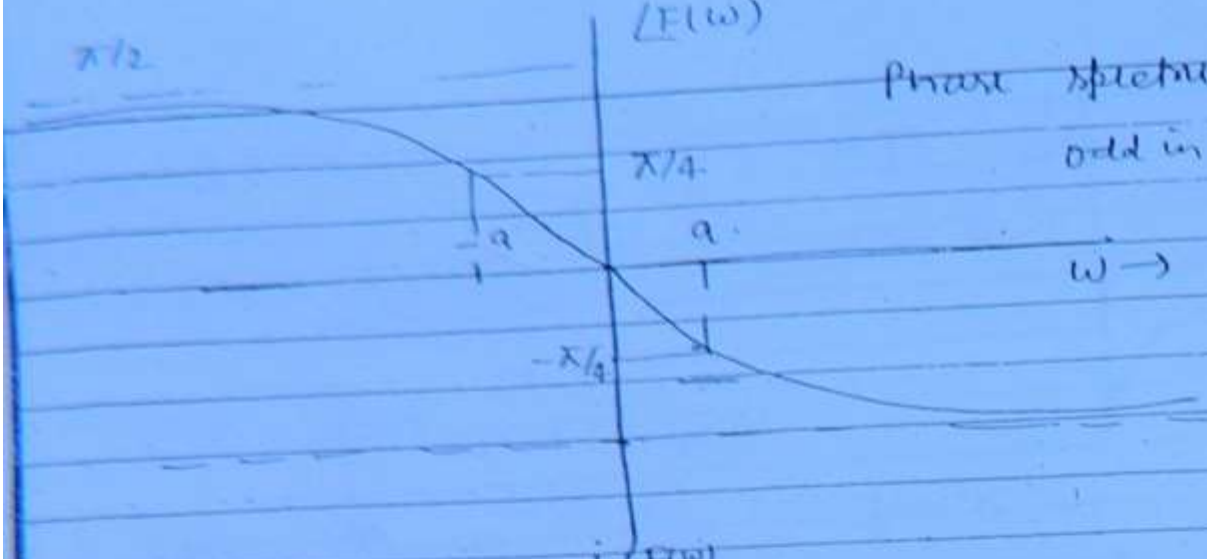


$$F[\mathrm{sgn}(t)] = F_{\substack{lt\\a\to0}}[g(t)]$$

$$= \lim_{a\to0} F[g(t)]$$

$$= \lim_{a\to0}\left[\frac{1}{a+j\omega} - \frac{1}{a-j\omega}\right]$$

$$F[\mathrm{sgn}(t)] = \frac{2}{j\omega}$$



Phase spectrum

odd in nature

$$F(\omega) = |F(\omega)|\, e^{j\angle F(\omega)}$$

$e^{at}\, u(-t)$

$$F(\omega) \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt$$

$$\boxed{F(\omega) = \frac{1}{(a-j\omega)}} = \underset{\text{even}}{\underset{\downarrow}{\frac{a}{a^2+\omega^2}}} + j\,\underset{\text{odd}}{\underset{\downarrow}{\frac{\omega}{a^2+\omega^2}}}$$

$$\left.\begin{array}{l} f(t) \longrightarrow F(\omega) \\ f(-t) \longrightarrow F(-\omega) \end{array}\right\}$$ time reversal Property

$F(t)\quad e^{t}_{u(-t)} + e^{-t}\, u(t)$

$$= \frac{1}{1-j\omega} + \frac{1}{1+j\omega}$$

if f(t) is real & even

$$\overset{F(\omega)}{\cancel{f(t)}} = \frac{2}{(1+\omega^2)} \longrightarrow$$ real & even

$$f_1(t) \longrightarrow F_1(\omega)$$

$$f_2(t) \longrightarrow F_2(\omega)$$

$$a f_1(t) + b f_2(t) \longrightarrow a F_1(\omega) + b F_2(\omega)$$

Q. Fourier Transform of $f(t) = e^{-at} u(t)$

$$= \int_0^{\infty} e^{-at} e^{-j\omega t} \, dt$$

$$= \frac{1}{-(j\omega + a)} \left[0 - 1 \right]$$

$$\boxed{F[\omega] = \frac{1}{a + j\omega}} = \frac{a - j\omega}{(a^2+\omega^2)} = \frac{a}{a^2+\omega^2} - j\frac{\omega}{a^2+\omega^2}$$

generally complex in nature

$$F_R(\omega) = \frac{a}{a^2+\omega^2}$$

$$F_R(-\omega) = \frac{a}{a^2+\omega^2} = F_R(-\omega) \rightarrow$$ even in nature

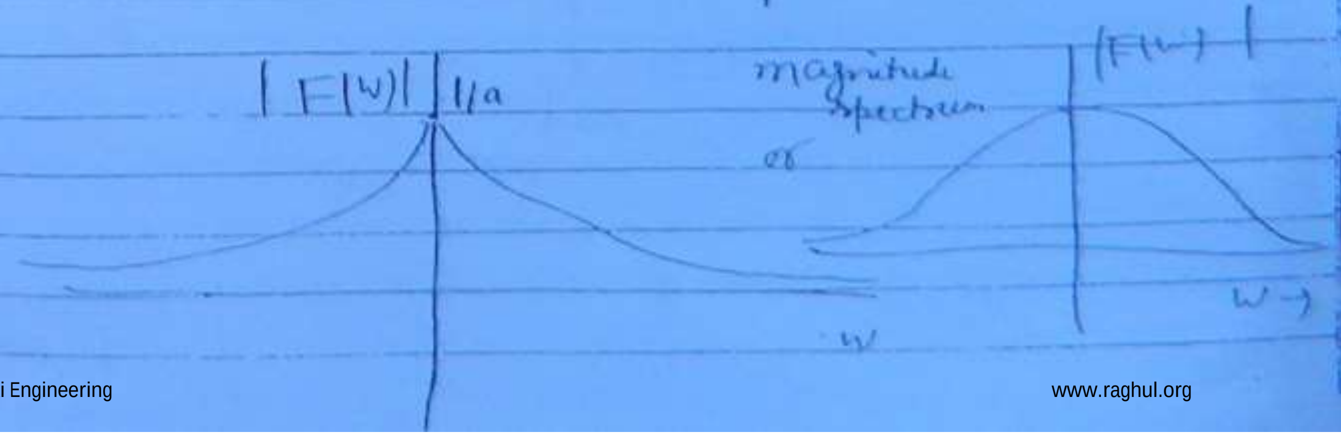
$$F_I(\omega) = \frac{-\omega}{a^2+\omega^2}$$

$$F_I(-\omega) = \frac{+\omega}{a^2+\omega^2} = -F_I(\omega)$$ odd in nature

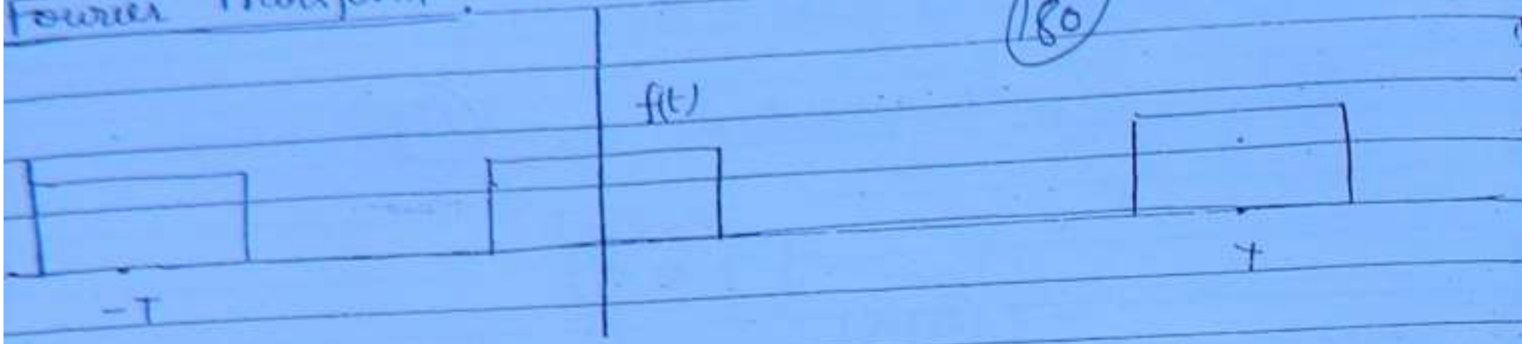
$F(\omega)$ is even conjugate or conjugate symmetric in nature

$$F(\omega) = F^*(\omega)$$

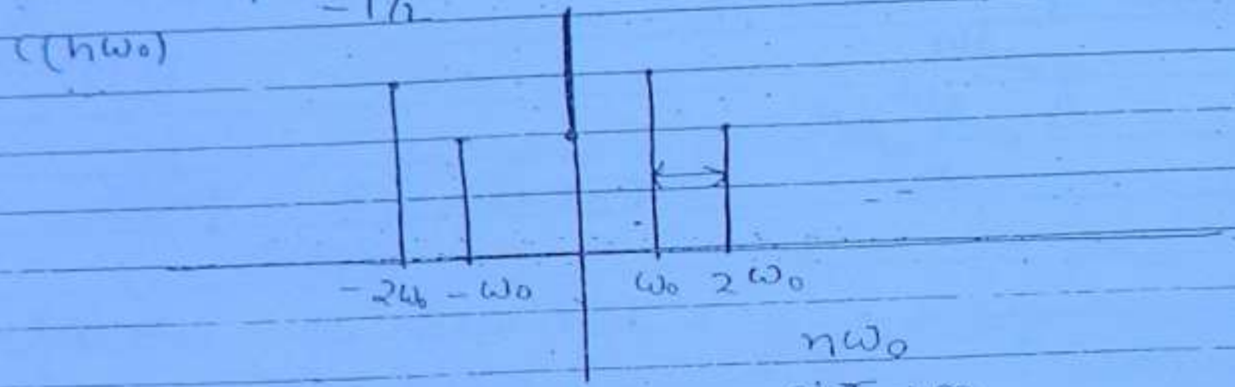
$$F(\omega) = |F(\omega)| \angle F(\omega) = \frac{1}{\sqrt{a^2+\omega^2}} \angle -\tan^{-1}\omega/a$$ $\left(1 \; \frac{\omega}{(a^2+\omega^2)}\right)$



Fourier Transform :→



$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt$$



$n\omega_0$

as $T \to \infty$

$n\omega_0 \longrightarrow \omega$

↓

Spacing becomes zero, means become Continuous

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$F[n\omega_0] = \int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$TC_n =$$

$$f(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty} F[n\omega_0]\, e^{jn\omega_0 t}$$

$$\boxed{\text{as } F[\omega] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \quad T \to \infty}$$

↓

as $T \to \infty$ f(t) becomes aperiodic signal

$$f(t) = \frac{\omega_0}{2\pi}\sum_{n=-\infty}^{\infty} F[n\omega_0]\, e^{jn\omega_0 t} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d$$

Q. $f(t) = \sum_{n=-100}^{100} \cos n\pi \, e^{jn\pi/100 t}$

$C_n = \cos n\pi \quad n \rightarrow -100 \text{ to } 100$

$\quad\;\; 0 \quad \text{other } n,$

$c_1 = \cos\pi$, ~~cos~~ $c_{-1} = \cos\pi$

$C_n = C_{-n}^{*}$

$C_n = C_{-1} \rightarrow \overset{\text{real}}{\text{even}}$ $\qquad f(t) = \text{real \& even}$

$f(t) = \sum_{-100}^{100} j \sin n\pi/2 \, e^{jn\pi/100 t}$

$C_n = j \sin n\pi/2 \quad n \rightarrow -100 \rightarrow 100$

$C_1 = j$

$C_{-1} = -j$

$C_1 = C_{-1}^{*}$ imaginary & odd

real & odd $\qquad f(t) = \sum_{n=-\infty}^{\infty} C_n e^{j}$

Q. A signal f(t) as fourier coefficient C_n find the fourier coefficient of

$f(t) + f(t-1) = C_n + fn \; f($

$f(-t$

$f(1-t) = f(-t+1)$

$= \cancel{C_{-n} e^{-jn\omega_0}}$

$= C_{-n} e^{-jn\omega_0}$

$f(t) + f(1-t) = C_n + \;(\quad e^{-jn\omega_0}$